new technologies, for understanding global economies, or for just figuring out the best deal on a loan. Yet, many people think that the world of mathematics is closed to them because they just don’t have a “math brain.” How many of your friends and family claim, maybe even joke, that they can’t do math? Yet, how many of those same people would light-heartedly say, “I can’t read”? Because our society accepts the myth that some people can and some people can’t do mathematics, we tend to give up early on children who are struggling with math, and then they give up on themselves. This belief is not only distressing, it is, quite simply, false.

Hopefully, a few of you have positive memories of mathematics and can imagine classrooms in which all students can participate and be successful; where mathematics is challenging, interesting, and makes sense; and where the learning environment allows students to participate without being afraid to ask questions or make mistakes. Even if this is not the mathematics you experienced as a child, it is not too late for you to learn mathematics in this way and to teach children mathematics so that they come to see themselves as confident and capable mathematicians.

The aim of this book is to help prospective and current teachers create a vision of learning and teaching mathematics in today’s classrooms. As you begin to understand the mathematics that you are or will be teaching, your confidence will grow in yourself as both a teacher and as an ongoing learner of mathematics. However, instilling new confidence may mean challenging your old assumptions about what mathematics is, how students learn mathematics, and how, as a teacher, you can support your students’ learning. The prevalence of negative images and experiences in mathematics in our society is one important reason for changing the ways in which mathematics is being taught. Another important reason for change is the mathematical needs of today’s society.

What Are Your Memories of School Mathematics?

“I always got teased for being a geek because I got As in math. I hated it and sometimes didn’t try as hard as I should.” —Pratima

“For me, math was boring. The teacher would write the new concept on the board with three examples, then we’d open the textbook and do all the odd questions. We’d work at our desks silently until the end of math class and then we’d finish the rest for homework. The next day we would do it all over again.” —Rhea

“I loved math all through my schooling. I liked that you could get a right answer to every question. I liked being the first one done and I liked being able to get 100%.” —Zobra

“I remember learning to count and add using blocks and pennies and counters. Math was fun in elementary school.” —Sophia

“I decided early on that I just didn’t have the brain for math. It didn’t seem to matter how hard I tried, I could just never catch on.” —Lucas

“I was good at math up to junior high and then after that it was all downhill. I kept falling farther behind. At some point I gave up and thought that everything we were doing was useless to me.” —Martin

Do any of these quotes remind you of your experiences in school? While some adults have fond memories of school mathematics, unfortunately, for many others, the learning of mathematics brings back feelings of frustration, anxiety, fear, and boredom. Every day we see evidence of the importance of mathematics in society through
Twenty-First Century Knowledge and Skills

At the beginning of the last century, mathematics content in schools focused on helping students gain mastery of addition, subtraction, multiplication, and division to solve everyday problems. The learning outcomes for students in their early teen years (the end of compulsory schooling in many provinces) emphasized occupation-specific number and geometry problems that might be relevant to merchants, salesmen, carpenters, farmers, homemakers, and others. A primary learning outcome was to compute problems involving money and measurements as efficiently and accurately as possible.

To participate meaningfully in the twenty-first century, students need to know and understand mathematics differently than their great-great-grandparents did. Accurate and efficient calculations for repetitive tasks are available at the touch of a button on every calculator, computer, cell phone, and mobile device. Mathematical skills are needed most when we are faced with new questions and novel problems. We use mathematics to recognize the issues, understand the problems, synthesize the available information, analyze the results, and communicate solutions in order to make decisions.

Many of the issues faced in today’s world cannot be solved by one person working in isolation. Creativity and innovation in response to complex problems require collaboration and communication among people with diverse experiences, skills, and perspectives. In the past, our primary means of communicating was through printed materials (e.g., letters, books, newspapers), face-to-face interactions, and telephone. Our methods of communicating have changed dramatically over the past two decades. We now have many more options using a range of electronic means, including email, video calling, wikis, blogs, podcasts, and so on. It is unimaginable what further enhancements to the ways we communicate in the next two decades might bring.

Classrooms, then, need to reflect the skills needed today and tomorrow: problem solving, critical thinking, creativity, collaboration, communication, and social responsibility. Not only are these skills essential to learning mathematics, but teaching mathematics through these general processes can also support children across all subject areas. Rather than textbooks or worksheets as their sole source of materials, students need opportunities to tackle interesting projects where they are expected to gather information from multiple sources, work creatively and collaboratively, and communicate their thinking with others. For example, when a guest speaker came in to share his experience of teaching in Uganda with Ms. Jacobs’s Grade 2 students, he mentioned that many of the people there did not have running water and had to collect rain water and carry water from a source several kilometres away. The children wondered how much water they carried, how heavy it would be, and how far they had to walk. Their questions led to many different investigations. For example, through Internet sources they learned that Canadians use an average of 340 litres of water each day in their homes. The children collected 2L and 4L milk jugs to help them imagine how much water that was. They calculated combinations of 2L and 4L jugs to total 340 litres. And they filled jugs with water to help them imagine having to carry water over long distances. They kept a class journal of all of their discoveries to share with other classes and parents.

Today’s teachers must use many sources and different approaches to meet the needs of all students and help them learn how to learn. It’s an exciting time to be a teacher: You have an opportunity to learn alongside your students.

Diversity in Today’s Classrooms

Learning mathematics today is about sharing diverse perspectives, about bringing forth new ways of knowing, and about different ways of understanding. Fortunately, today’s classrooms are significantly more diverse than they were even a decade or two ago. Students represent a range of cultures, languages, abilities, and socioeconomic circumstances. Teachers who believe that all students are capable of learning mathematics and contributing to the learning of others can capitalize on diversity in the classroom for the benefit of all students.

Given the strong governmental commitment to equity, access, and inclusion in Canadian schools, teachers are expected to attend to the different learning, language, and cultural needs of all students. Of particular interest and concern are students from First Nations, Métis, and Inuit (FNMI) families. The expectations for learners in typical mathematics classrooms often conflict with the oral, relational, contextual, and holistic ways of Indigenous knowing (Sterenberg et al., 2010). For example, Inuktitut, an official language in Nunavik, Nunavut, and Northwest Territories has multiple oral forms for number words, depending on the context that is used, such as pingsasut (three objects), pingasut (three objects), pingasunnaartitut (a group of three), and pingasunnaarqisimmam (a pattern of three) (Poirier, 2007). Asking children to count to 10 without a context for what is being counted may be meaningless. Developing knowledge and appreciation of the traditions, cultures, and perspectives of these students, finding ways to make connections to the experiences of these children, and allowing them to contribute to the learning of others is a challenging but vital task.

Students who have recently immigrated to Canada or who speak a different language at home may also contribute to the cultural, linguistic, and educational diversity in the
classroom. If we consider the example of counting again, number names are easier to remember in some languages more than others. In English speaking countries, numbers such as two (2), twelve (12), and twenty (20) are seemingly unrelated when said orally. In Mandarin, two is èr, twelve is shí èr (ten-two) and twenty is èr shí (two-ten), making it much easier for children to remember number names and understand their relational value. Educators must provide opportunities for children to develop proficiency in the language of their schooling, but the learning of all students can be enhanced when teachers create space for sharing and learning from the cultural knowledge students bring to school.

While all students require educational activities pertinent to their needs, different resources, activities, learning environments, and assessment practices are often necessary for learners with behavioural, communicational, cognitive, or physical exceptionalities or special needs. For children who are visually impaired, learning to count poses significant challenges as they must rely on sound and touch to develop a sense of “how many” (Figure 1.1). The abacus has long been used to teach early numbers to these students, and interestingly, may lead to a deeper understanding of number relationships and place value compared to paper and pencil methods. Various forms of abaci are now making their way into many classrooms to help all children with number knowledge.

All children are unique. They learn and think differently. Regardless of the unique abilities of some students and the possible challenges they face while learning mathematics, teachers must be persistent and remain firm in their belief that all students can learn mathematics and contribute to the learning of others. We no longer expect all children to produce identical answers using the same procedures, strategies, and formulas. Today’s mathematics allows children to communicate their diverse ways of thinking and doing mathematics. Consider Matthew’s work on the classroom task “Today’s number is 24” (Figure 1.2). Matthew is a Grade 2 student identified as gifted in his educational plan. What can we learn about Matthew’s thinking about numbers that we wouldn’t have learned by asking questions such as: 18 + 6 = ? How might Matthew’s thinking contribute to the diverse ways of thinking and doing mathematics in the classroom?

Teaching mathematics means setting high expectations for every student in the classroom. Rather than seeking ways to reduce or eliminate diversity in culture, language, knowledge, experience, and perspective, mathematics curriculum and teaching approaches today provide opportunities to benefit from diversity through the sharing of multiple insights to learn mathematics content and to think mathematically. Diversity can contribute to a rich learning environment where differences in thinking are valued and teachers can use the experiences students bring to the classroom to the benefit of all learners.

**Mathematics Curriculum in Canada**

It is common to think that the same mathematics has always been taught in exactly the same way for a hundred years. But mathematics education across North America has a rich history and is constantly undergoing change. The past two
decades have brought significant reforms to mathematics education that reflect the technological and informational needs of society, recent research on how students learn and understand mathematics, the importance of providing access and opportunity for all students, and lessons learned from international approaches and research on mathematics teaching and learning.

Curriculum documents are often what drive change in classrooms by specifying what mathematics is valued and how teachers are expected to support student learning. Each province and territory in Canada has its own educational ministry that is responsible for setting expectations for learning by producing a curriculum document. Revisions to curriculum take place through collaboration among many stakeholders including teachers, school and government administrators, post-secondary educators, parents, and community members. Within the past decade, the Yukon, the Northwest Territories, and Nunavut, and eight of ten provinces including British Columbia, Alberta, Saskatchewan, Manitoba, Prince Edward Island, New Brunswick, Nova Scotia, and Newfoundland & Labrador have adopted and subsequently adapted the *Western and NorthernCanadian Protocol Common Curriculum Framework for K–9 Mathematics* (Alberta Education, 2006). While each educational ministry has made changes to the original WNCP-CCF, such as to the learning outcomes or terminology, much of the organization, content, processes, and goals remain unchanged. (Throughout this text, we will refer to the WNCP, but be sure to check your regional curriculum document for variations.) The two provinces with substantially different mathematics curriculum documents are Ontario (Ontario Ministry of Education, 2005) and Quebec (Gouvernement du Québec Ministère de l’Éducation, 2001; 2004). These latter two curricula were developed within their respective provinces, and while their formats are different from those based on the WNCP, all the Canadian curriculum documents highlight similar goals for students, mathematics content, and mathematical processes.

### Goals for Student Learning

Each provincial and territorial curriculum document lists or describes the main goals and outcomes for learning mathematics, including gaining mathematical understanding, learning process skills such as communication and reasoning mathematically, and developing a positive disposition towards mathematics. While the particular set of goals is different across the documents, the development of a positive disposition towards mathematics is a consistent theme. Students are to exhibit curiosity, risk taking, self-confidence, and perseverance. They are expected to become mathematically literate and gain an appreciation for mathematics in society.

While it might seem odd to place such a strong emphasis on having students display positive dispositions towards mathematics, we know that learning is inhibited by fear, anxiety, and poor self-confidence. Having confidence in one’s own ability to do and make sense of mathematics has a significant impact on learning. Ensuring that all students believe that they are capable of learning mathematics is an important objective.

Part of promoting positive and productive dispositions means challenging some long-held notions about what it means to be successful in mathematics. Someone who is good at mathematics isn’t necessarily the fastest and/or the one who is always right. Someone who is mathematically minded is curious, motivated to learn, and confident in her abilities to work through uncertainties; reflects back on both correct and incorrect approaches to a task; and is willing to take risks which might mean asking questions or trying new strategies. Perhaps most importantly, a key characteristic of a successful mathematician is persevering on challenging tasks and through unfamiliar problems.

Teachers and parents are instrumental in helping children develop positive dispositions towards mathematics. As they work alongside children, they too can demonstrate a willingness to explore, take risks, and display curiosity in both the mathematics the children are learning and in the children’s responses. The parent or teacher who says, “I was never any good in math either” perpetuates the myth that not everyone is capable of learning mathematics.

Another disposition emphasized in today’s curricula is an appreciation of the role of mathematics in human history and its contributions to science, technology, philosophy, and art. Mathematics has played a part in many scientific and technological innovations that have made our lives more comfortable and convenient. However, some of these advances may have occurred at the expense of the planet’s ecological balance. In addition to appreciating mathematics in society, there is also a place for viewing mathematics from a critical perspective. For example, mathematical models are often used to predict the environmental impact of projects. Decisions to proceed with a project are also based on mathematical cost versus benefit calculations. An aim of mathematics instruction is to help students attend to and question decisions that are justified using mathematics—especially when those decisions affect living beings and the environment.

### Mathematics Content

There is significant commonality across all provinces as to what mathematics is presently believed to be important for students from kindergarten to Grade 8. Learning expectations are organized into content areas called *strands* or *organizers* and further into substrands or suborganizers. Although the labels are slightly different, mathematics content across Canada can be organized into five content areas described here as number and number sense; patterns and algebra; measurement; geometry and spatial sense; and data analysis and probability (see Table 1.1).
The curriculum documents and resources provided by the educational ministry in your province or territory at the following websites:

Alberta: http://education.alberta.ca/teachers/program/math/educator/progstudy.aspx
British Columbia: http://www.bced.gov.bc.ca/irp/welcome.php
Manitoba: http://www.edu.gov.mb.ca/k12/cur/math/
New Brunswick: http://www教育资源.nb.ca/0000/anglophone-e.asp
Ontario: http://www.edu.gov.on.ca/eng/curriculum/elementary/math.html
Nova Scotia: http://www.ednet.ns.ca/
Prince Edward Island: http://www.progetudes.gov.sk.ca/
Quebec: http://www.mels.gouv.qc.ca/enseignants/programmes-detudes/
Saskatchewan: http://www.progetudes.gov.sk.ca/


Number and number sense refer to concepts and skills needed to develop an understanding of and operations with whole numbers, integers, fractions, decimals, ratios, and percent.

Patterns and algebra include the study of repeating and growing patterns and the ways in which relationships and regularities in patterns and number properties can be modelled using tables, graphs, and algebra.

Geometry and spatial sense include the ability to recognize, represent, construct, transform, and describe relationships among two- and three-dimensional objects.

Measurement introduces students to the measurable attributes of objects and the nonstandard and standard units and processes involved in measurement. Measureable attributes include length, perimeter, area, volume, capacity, mass, and time.

Data analysis and probability involve the different ways one can organize, display, analyze, and interpret data collected through observation, database, survey, experiment, or other means.

Each content area includes both general and specific learning outcomes or expectations for each grade. Although the same strands or content areas usually apply across grades K through 8, number and number sense is more heavily emphasized from kindergarten through Grade 5, and algebra content becomes more prominent in the middle grades. Section II of this book (Chapters 8 through 23) provides extensive detail of the mathematics, the curriculum expectations, children’s learning, and pedagogical approaches in each of the content areas.

### Mathematical Processes

While the mathematics content describes what students are to learn, the mathematical processes describe how that learning should take place (see Table 1.2). The mathematical processes, as they are referred to in most curriculum documents, are the interconnected process skills that utilize students’ multiple ways of knowing and learning mathematics.

### TABLE 1.2

<table>
<thead>
<tr>
<th>Western Provinces, Atlantic Provinces, and Territories</th>
<th>Ontario Processes</th>
<th>Quebec Competencies and Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning</td>
<td>Reasoning and proving</td>
<td>Uses mathematical reasoning</td>
</tr>
<tr>
<td>Communication</td>
<td>Communicating</td>
<td>Communicates using mathematical language</td>
</tr>
<tr>
<td>Problem Solving</td>
<td>Problem solving</td>
<td>Solves a situational problem</td>
</tr>
<tr>
<td>Connections</td>
<td>Connecting</td>
<td>Makes connections</td>
</tr>
<tr>
<td>Mental Mathematics and Estimation</td>
<td>Selecting [tools and computational strategies]</td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>Selecting [tools and computational strategies]</td>
<td></td>
</tr>
<tr>
<td>Visualization</td>
<td>Reflecting</td>
<td>Representing</td>
</tr>
</tbody>
</table>

Again, while they are labelled differently, there is consistency in four key processes across Canadian curricula: reasoning, communication, problem solving, and connections.

Reasoning emphasizes logical thinking, helping us decide if and why our answers make sense. Reasoning includes identifying similarities and differences, forming generalizations, making and testing conjectures and predictions, reaching conclusions based on evidence, and using reasoning to justify, verify, and prove solutions by building upon what is already known or assumed to be true. Students develop the habit of providing a rationale as an integral part of every answer.

Communication includes reading, writing, drawing, manipulating, listening to, discussing, and explaining mathematical ideas. Communication also includes the proper use of symbols, vocabulary, terms, conventions, and notations in oral, visual, and written form. Learning to communicate in mathematics fosters interaction and the exploration of ideas in the classroom, as students learn through active discussions about their thinking. No better way exists for deepening one's understanding of an idea than to attempt to articulate it to others.

Problem solving is an intellectual and creative process developed by and through reasoning. Not knowing where or how to begin is the essence of problem solving. It is through problem solving that a student builds on previous experiences and learns new mathematics. Problem solving is often considered a primary focus and goal of mathematics. Problem solving is a significant role in art, science, language arts, and social studies. Students develop the habit of providing a rationale as an integral part of every answer.

Connections are the means by which mathematics content can be seen as useful, relevant, and integrated. Students need opportunities to see how mathematical ideas connect to one another in a useful network of connected ideas. Mathematics should also be connected to the real world and to other disciplines. Students should see that mathematics plays a significant role in art, science, language arts, and social studies. This suggests that mathematics should frequently be integrated with other discipline areas and that applications of mathematics should be explored in real world contexts.

In addition to these four common processes, there are several other related processes identified in provincial curricula:

Mental Mathematics and Estimation (WNCP): “Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating” (Alberta Education, 2006, p. 7).

Reflecting (Ontario): “Good problem solvers regularly and consciously reflect on and monitor their own thought processes” (Ontario Ministry of Education, 2005, p. 14). Through reflection on both incorrect and correct solutions, students can consider alternative strategies, whether or not an answer seems reasonable, or why a problem was easy or difficult.

Representing (Ontario): “In elementary school mathematics, students represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words, and symbols. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret realistic problem situations” (Ontario Ministry of Education, 2005, p. 16).

Selecting tools and computational strategies (Ontario): “Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems” (Ontario Ministry of Education, 2005, p. 14).

Technology (WNCP): “Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures and solve problems” (Alberta Education, 2006, p. 9).

Visualization (WNCP) is the process of thinking in images in order to compose, decompose, or transform mathematical concepts. Visualization is an important tool for many areas of mathematics content, including number, pattern, geometry, and data analysis.

These processes should not be regarded as separate content or strands in the mathematics curriculum; rather, they direct the methods of doing all mathematics and, therefore, should be seen as integral components of all mathematics learning and teaching. In provincial curriculum documents based on the WNCP, each learning outcome identifies the relevant processes in square brackets at the end of each outcome. For example, in Grade 2, students are expected to “demonstrate if a number (up to 100) is even or odd [C, CN, PS, R]” (Alberta Education, 2006, p. 18). As students engage in activities related to the outcome, they should also be developing process skills of communication (C), connections (CN), problem solving (PS), and reasoning (R). In Ontario, the Mathematical Process Expectations for all seven of its processes are described as part of the introduction to each grade and are meant to be integrated across all content strands. Finally, in Quebec the competencies and making connections are an integral part of the entire program. In this text, mathematical processes are fundamental to all aspects of mathematics teaching and learning. Processes are a part of every task in this text—even when they are not explicitly identified.

Knowing the organization of mathematics content, mathematical processes, and goals in your curriculum document is important, but teaching mathematics today will
likely require that you learn the content using the intended processes and strive towards the goals alongside your students.

**An Invitation to Learn and Grow**

Someday soon you will find yourself in front of a class of students, or perhaps you are already teaching. As a prospective or practising teacher facing the challenge of teaching in the twenty-first century, you may be required to confront some of your personal beliefs—beliefs about what it means to *do mathematics*, how one goes about *learning mathematics*, how to *teach mathematics*, and what it means to *assess mathematics* integrated with instruction. What general ideas will guide the way you will teach mathematics? How will you foster students’ positive dispositions and their understanding of mathematics content and processes for thinking mathematically?

This chapter presented the beginnings of a vision of mathematics learning and teaching that is currently embedded in curricula and teaching resources across Canada. There is no recipe for effective mathematics teaching, but there are research-based strategies that have been shown to support children’s learning of mathematics in ways that are far superior to memorization and drill activities. By becoming informed, you will make stronger teaching decisions and will create valuable and purposeful learning opportunities for your students. The information, activities, questions for reflections, recommendations for further reading, and online resources in this text provide multiple opportunities for you to think critically and openly about mathematics teaching. Your knowledge of mathematics and how children learn mathematics is the most important tool you can acquire to be an effective teacher of mathematics.

As you learn new research-based strategies, it is important to continue to reflect on your past experiences and the impact they have had on your own disposition towards learning and teaching mathematics. Teachers, consciously or not, often pass on their attitudes and beliefs about a content area onto their students. If mathematics is a subject you enjoy, students can be swept up in your enthusiasm and they will experience mathematics as a relevant and interesting subject.

From this point forward, stop accepting people’s claims that they can’t do math … especially when those words are spoken by a child or in the presence of children.

**Becoming a Teacher of Mathematics**

Consider your past experiences in mathematics, your own disposition towards learning and teaching mathematics, and your current knowledge and skill in mathematics content and mathematical processes. The next section lists and describes the characteristics, habits of thought, skills, and dispositions you will need to succeed as a teacher of mathematics. Which of these areas are strengths for you and which do you see as challenges?

**Mathematical Disposition.** A key responsibility of the teacher is to foster positive dispositions towards mathematics. One of the best ways to do this is by modelling these dispositions yourself by showing genuine curiosity in the mathematics your students are engaged in and in their responses, strategies, and solutions; by seeking out possible connections between approaches and thinking through unfamiliar problems with confidence; by being willing to look for more efficient approaches or to generalize findings to other problems and tasks; and by drawing attention to the beauty of mathematics and its applications in society.

Being successful in mathematics is not about already knowing how to answer a question or solve a problem. It’s about becoming curious, engaging in inquiry, and continuing to persevere even when (indeed, especially when) you don’t know an answer. As Einstein said, “It’s not that I’m so smart, it’s just that I stay with problems longer.” As you move through this book and work the problems yourself, you will learn strategies that will help you anticipate challenges and identify ways to support your students’ learning. As you experience the material in this book, if you ponder, struggle, talk about your thinking, and reflect on how it all fits or doesn’t fit your previous experience, then you enhance your repertoire as a teacher. As you model these characteristics to your students, they too will begin to value persistence over speed or correctness. Creating opportunities for your students to struggle is part of teaching and learning (Stigler & Hiebert, 2009).

**Mathematical Understanding.** The mathematics that a student of mathematics or even a mathematician needs to know is qualitatively different from the mathematics that teachers need to know. As a student, your goal is to compress and streamline processes, procedures, and proofs, but teachers are required to do the opposite. They need to “unpack” the mathematics (Ball & Bass, 2003).

Suppose you wanted to assess student understanding of ordering decimals. Which of the following lists of numbers, (a), (b), or (c) would provide you with the evidence you needed to perform an accurate assessment?

<table>
<thead>
<tr>
<th>(a)</th>
<th>.5</th>
<th>7</th>
<th>.01</th>
<th>11.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>.60</td>
<td>2.53</td>
<td>3.14</td>
<td>.45</td>
</tr>
<tr>
<td>(c)</td>
<td>.6</td>
<td>4.25</td>
<td>.565</td>
<td>2.5</td>
</tr>
</tbody>
</table>

*Source: Ball & Bass, 2003, p. 9.*

Knowing which list of numbers to choose requires a different kind of reasoning than the reasoning needed to do the task as a student. A teacher needs to know what mathematical understanding is needed to order decimals.
successfully and to know what difficulties students might experience in trying to do so. It might not be immediately obvious which list to choose, so try ordering the numbers in each row from smallest to largest without paying attention to the placement of the decimal point. Now, put the decimal back in, and order the numbers from least to greatest again. In two of the three lists, the order remains the same, regardless of whether or not you paid attention to the placement of the decimals; neither of these two lists would be useful for assessing understanding of ordering decimals. Only (c) would be a valid assessment of a student’s ability to order decimals correctly.

Or consider the question, “What is multiplication?” As a teacher, you need to be aware of the different images and actions involved in multiplication and recognize that different interpretations are experientially distinct from one another and not applicable to every situation. For example, thinking of multiplication as repeated addition doesn’t make a lot of sense for something like $\frac{1}{2} \times \frac{1}{3}$. Davis & Simmt (2006) asked elementary and middle school teachers, “What is multiplication?” and the ensuing discussion led to many different images, definitions, and analogies. For example, the teachers described multiplication as repeated addition (e.g., $3 \times 4 = 4 + 4 + 4$); as an array or grid (e.g., 3 rows with 4 in each row); as a measurement of area or volume (e.g., a rectangle that is 3 cm by 4 cm is 12 cm²); as a ratio or rate (e.g., 3 kg for $\frac{4}{4}$kg is $\frac{12}{4}$); etc.

Designing tasks and choosing useful interpretations of a concept are just two teacher-specific tasks that require a different kind of mathematical understanding (see Table 1.3 for others).

Teaching children and youth mathematics requires you to have a coherent, evolving, flexible, and adaptive understanding of mathematics (Davis & Renert, 2013). Rather than expecting every student to do as you do, you will be encouraging multiple ways of interpreting mathematical concepts and inquiries. To respond to students’ curiosity, uncertainties, questions, and frustrations, you may need to rework mathematical concepts that you thought you knew, and move among different but interconnected interpretations of concepts by drawing on images, analogies, and metaphors. As a result, you will be able to respond more flexibly to students, allowing you to extend their understanding.

**Professional Learning.** Make time to be self-aware and reflective. No matter whether you are a preservice teacher or an experienced teacher, there is always more to learn about the content and methodology of teaching mathematics. The ability to examine oneself for areas that need improvement or to reflect on successes and challenges is critical for professional growth and development. Take the time to seek out professional learning opportunities. It is never too early—or too late. Most provinces have mathematics special interest groups associated with their provincial teachers’ association who provide information, host professional development events, and publish journals or newsletters. The following groups have information online.

- British Columbia Association of Mathematics Teachers (bcamt.ca)
- Mathematics Council of the Alberta Teachers’ Association (mathteachers.ab.ca)
- Saskatchewan Mathematics Teachers’ Society (smts.ca)
- Manitoba Association of Mathematics Teachers (mamt.mb.ca)
- Ontario Association for Mathematics Education (oame.on.ca)
- Groupe des responsables en Mathématique au Secondaire (grms.qc.ca)
- Association Mathématique du Québec (archimede.mat.ulaval.ca/amq/)
- Nova Scotia Mathematics Teachers Association (mta.nstu.ca)

Another outstanding source for professional learning is the National Council of Teachers of Mathematics (NCTM; www.nctm.org). NCTM, with headquarters in Reston, Virginia, is the world’s largest mathematics education organization with more than 90,000 members. Many of the provincial associations named above are affiliate members of NCTM and there is Canadian representation on the NCTM Board of Directors and its many committees. NCTM offers professional development opportunities, produces and sells educational materials, edits and prepares research and professional journals, and carries out many other initiatives that contribute to mathematics education across North America and the world. You will see many references to NCTM publications throughout this book. NCTM has been one of the most prominent influences on Canadian mathematics curriculum over the past 30 years.

The best teachers are always trying to grow and expand their practice through the latest article, the newest book, the most recent conference, or by signing up for the next

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**TABLE 1.3**

<table>
<thead>
<tr>
<th>FREQUENT TASKS OF TEACHING</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Posing mathematical questions</td>
</tr>
<tr>
<td>• Giving and appraising explanations</td>
</tr>
<tr>
<td>• Choosing or designing tasks</td>
</tr>
<tr>
<td>• Using and choosing representations</td>
</tr>
<tr>
<td>• Recording mathematical work on the board</td>
</tr>
<tr>
<td>• Selecting and sequencing examples</td>
</tr>
<tr>
<td>• Analyzing students’ errors</td>
</tr>
<tr>
<td>• Appraising students’ unconventional ideas</td>
</tr>
<tr>
<td>• Mediating a discussion</td>
</tr>
<tr>
<td>• Attending to and using math language</td>
</tr>
<tr>
<td>• Defining terms mathematically and accessibly</td>
</tr>
<tr>
<td>• Choosing or using math notation</td>
</tr>
</tbody>
</table>

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The best teachers never “finish” learning nor exhaust the number of new connections that they make, and, as a result, they never see teaching as stale or stagnant. So, as John Van de Walle said with every new edition, “Enjoy the journey!”

**RESOURCES for Chapter 1**

**Recommended Readings**

The following articles and books focus on the key themes of Chapter 1: Positive dispositions, teaching mathematics to diverse students, twenty-first century learning, and curricula across Canada.

**Articles**


In this article, Alison Gear, an early-learning coordinator, shares her experience of kindergarten students and parent participation in a culturally responsive program that includes lessons based on traditional Haida Nation stories in British Columbia.


This article from mathematicians and mathematics educators across Canada presents a discussion initiated in a study group at the Canadian Mathematics Education Forum in 2009. Their discussion addresses the following questions in narrative form: “What do Indigenous knowledges offer for teaching and learning mathematics by both Aboriginal and non-Aboriginal students and teachers? What role does language play? What role does place, community, and culture play?” This article was also reprinted in vinculum: Journal of the Saskatchewan Mathematics Teachers’ Society, *2*(1) in 2010.


This article provides a simple six-question survey to assess students’ attitudes and dispositions in math. Open-ended questions such as “to be good in math, you need to … because …” and “math is hard when …” can provide interesting insight into what students think of as important in mathematics and what they identify as their strengths and difficulties. The article also provides response samples and a sample analysis of Grade 4 student responses.

**Books**


**Online Resources**

**Math Forum**  
[mathforum.org](http://mathforum.org)

The Math Forum at Drexel University in Philadelphia is possibly the best one-stop shop for everything mathematics on the Internet. *Ask Dr. Math* is a huge database of questions and answers to mathematical problems and questions. The site also has many resources for teachers, including problems of the week, lessons, and a forum for asking questions about teaching mathematics.
Chapter 1 Teaching and Learning Mathematics in the Twenty-First Century

Math Central
mathcentral.uregina.ca
Math Central is a website maintained by the University of Regina. It includes teaching resources, glossaries, a place to search for or ask questions on mathematical problems, information about mathematicians in different careers, outreach activities, problems, and many other interesting sources of information.

National Council of Teachers of Mathematics
www.nctm.org
Here you can find out all about NCTM, its belief statements, and its positions on important topics. You will also find an overview of Principles and Standards and free access to interactive applets (see Standards—Electronics), membership and conference information, publications catalogue, links to related sites, and much more. Members have access to even more information.

Illuminations
illuminations.nctm.org
This companion website to NCTM, sponsored by NCTM and Marcopolo, provides lessons, interactive applets, and links to websites for learning and teaching mathematics.

Writing to Learn
At the end of each chapter of this book, you will find a series of questions under this heading. The questions are designed to help you reflect on the most important ideas of the chapter. Writing (or talking aloud with a peer) is an excellent way to explore new ideas and incorporate them into your own knowledge base. The writing (or discussion) will help make the ideas your own.

1. What mathematical skills and knowledge are needed for participating in today’s society? How and why are they different from the skills and knowledge that were needed by your parents or your grandparents when they were going to school?
2. Inclusion and diversity are key issues in today’s schools. What do you see as the advantages and disadvantages of teaching mathematics to students with diverse perspectives and diverse learning needs?

For Discussion and Exploration
1. Examine your provincial or territorial curriculum document. What are the goals for learning mathematics? What are the content areas and processes addressed? How are they defined?
2. Go online and find mathematics curriculum information from your educational ministry. In addition to the curriculum document, what other resources are available for teaching mathematics?
3. Most provinces have mathematics special interest groups associated with their provincial teachers’ association (see the websites listed in the final section of this chapter). What events, publications, and resources are available in your province or territory?
4. Interview one or two students in elementary or middle school. Ask them what they like and don’t like about mathematics. Ask them what resources they use to learn and do mathematics in their class (e.g., textbook, worksheets, interactive whiteboard tools, online applets, computer software, etc.). How are their experiences learning mathematics the same or different from yours?
5. Go to the National Council of Teachers of Mathematics (NCTM) site at nctm.org. Explore the Lessons & Resources tab for lessons, problems, and teaching tips. Now go to the Journals & Books tab and select one of the following professional journals: Teaching Children Mathematics (for elementary grades), Mathematics Teaching in the Middle School or Mathematics Teacher (for high school). Review the Free Preview article or locate an article of interest accessible through your library.

MyEducationLab™
Visit MyEducationLab to access an electronic version of the text, as well as a variety of topics that enhance the text material. The topics include the following to support your learning in the course:
- Assessment, including Building Teaching Skills and Dispositions and Video Assignments
- Discussion board questions
- Videos, simulations, activities, case studies, and other useful course resources