understand the ideas presented must focus their attention on following rules or directions in an instrumental manner (i.e., without a conceptual understanding). This, of course, leads to endless difficulties and limits understandings or leaves students in need of significant remediation. In this chapter we examine how we can attend to student differences through planning and teaching strategies. We begin by looking at differentiated instruction for all learners and then focus on strategies for students with more specific identified learning needs.

Differentiated Learning and Teaching

The students in our classrooms have different interests, experiences, cultural orientations, languages, and physical abilities. Differentiation is an approach to teaching and learning that includes proactive planning for differences in what students need to learn, how they will learn, and how they are expected to show what they learned. Providing for these differences allows you to maintain high expectations for students while also allowing for different ways of thinking and doing mathematics. In this section we will look at differentiation strategies in the mathematics classroom including tiered tasks, learning centres, choice boards, think-pair-share strategies, and graphic organizers.

Tiered Tasks

The criteria for a rich task include opportunities for variable responses to demonstrate a range of thinking (see Chapter 3). A tiered task pushes those rich tasks a bit further by defining different tiers of complexity and challenge based on the learning needs of the students in the classroom.

Bill has one more toy car than Megan. Fred has 1 less car than Megan. Megan has 5 toy cars. How many cars do Bill and Fred have?

“My students can’t solve word problems—they don’t have the reading skills.”

“I am not doing as many writing activities during math instruction because I have so many ELLs in my class.”

“The only success my students with cognitive disabilities have is with practice worksheets.”

Although we want students to experience success, we also need to maintain high expectations for students that involve doing mathematics, such as making conjectures and predictions, searching for and generalizing patterns, and verifying and justifying explanations. Every classroom contains students with a range of abilities and backgrounds. In the mathematics classroom today, the most important work of teachers is to plan and teach lessons that support and challenge all students. For this reason, one size does not fit all when it comes to planning, teaching, and assessment strategies (Tomlinson, 2005).

Diversity in thinking mathematically is something to be celebrated and cultivated, not eliminated (think back to Chapter 1). The sharing of different perspectives can benefit all learners. Interestingly, and perhaps surprisingly to some, the inquiry-based approach to teaching allows teachers to attend to the diverse range of students in their classrooms. In the inquiry-based classroom, children are making sense of the mathematics in their own ways, bringing to the tasks the skills and ideas that they know and understand. In contrast, in a traditional highly directed lesson, it is often assumed that all students will understand and use the same approach and the same ideas as determined by the teacher. Students not ready to understand the ideas presented must focus their attention on following rules or directions in an instrumental manner (i.e., without a conceptual understanding). This, of course, leads to endless difficulties and limits understandings or leaves students in need of significant remediation. In this chapter we examine how we can attend to student differences through planning and teaching strategies. We begin by looking at differentiated instruction for all learners and then focus on strategies for students with more specific identified learning needs.
Differentiated Learning and Teaching

The adaptation can involve any of the following (Kingore, 2006):

1. The degree of assistance, perhaps by providing additional examples, partnering students, or providing guided support to a small group of students.
2. The complexity of process. This includes how quickly the lesson is, how many instructions are given at one time, and how many higher-level thinking questions are included as part of the task.
3. The complexity of the task given. This can be accomplished by simply changing the numbers in a problem (i.e., making them smaller or larger) or by adding more difficult problems or applications to a task (e.g., see the Toy Car task below).
4. How structured the task is. Students with special needs, for example, may benefit from highly structured tasks with supports, but gifted students often benefit from a more open-ended structure where the student chooses the approach to use (e.g., see the Properties of Parallelograms task).

Pause AND REFLECT

Think of different types of learners (ELLs, students with special needs, gifted learners, unmotivated students). How does the adapted lesson above meet each of their learning needs?

The following example illustrates how to tier a task based on structure. Notice that the different tasks vary in how open-ended the work is, yet all three tasks focus on the same learning goal of identifying the properties of parallelograms.

Grades 1–2: Original Toy Car Task
Eduardo had 9 toy cars. Erica came over to play and brought 8 cars. Can you figure out how many cars Eduardo and Erica have together? Explain how you know.

The teacher distributes cubes to students to model the problem, and paper and pencil to illustrate and record how they solved the problem. He asks students to model the problem and be ready to explain their solution.

Grades 1–2: Adapted Toy Car Task
Eduardo had ____ toy cars. Erica came over to play and brought ____ cars. Can you figure out how many cars Eduardo and Erica have together? Explain how you know.

Choose which numbers to use. Challenge yourself!

A) 9, 8 B) 19, 28 C) 139, 228

The teacher asks students what is happening in this problem and what they are going to be doing. In each case, students must use words, pictures, models, or numbers to show how they figured out the solution. Various tools are provided (connecting cubes, counters, and hundreds charts) for their use.

By allowing students to choose the numbers in the adapted task, the teacher provides students with the opportunity to control the level of difficulty. Alternatively, you could just leave the blanks in the problem and have students choose their own numbers. In this task, teachers may need to provide initial support to help students make their number choices, but such tiered tasks encourage students to assess and work at their own level of learning. Follow-up discussions are still possible, since the content is the same and all children can feel as though they worked on the same task.

Grades 5–6: Properties of Parallelograms
Students are given a collection of parallelograms including squares and rectangles as well as nonrectangular parallelograms. The following tasks can be distributed to different groups based on their learning needs and prior knowledge of quadrilaterals:

- Open-ended: Explore the set of parallelograms. Measure angles and sides using your ruler and protractor. Make a list of the properties that you think are true for every parallelogram.
- Slightly structured: Use your ruler and protractor to measure the parallelograms. Record any patterns that are true for all of the parallelograms related to:

<table>
<thead>
<tr>
<th>Sides</th>
<th>Angles</th>
<th>Diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Highly structured: First, sort the parallelograms into rectangles and nonrectangles. Then, use a ruler to measure the sides and a protractor to measure the angles. Explore the parallelograms to find patterns and rules that define the shapes as parallelograms.

1. What pattern do you notice about the measures of the sides of all the parallelograms in the nonrectangle set? What pattern do you notice about the measures of the sides of all the parallelograms in the rectangle set?
2. What pattern do you notice about the measures of the angles of all the parallelograms in the nonrectangle set? What pattern do you notice about the measures of the angles of all the parallelograms in the rectangle set?

For many problems you can differentiate learning by using tiered tasks or by providing tasks with different levels of structure. Where possible, giving a choice increases student motivation and helps students become more self-directed learners (Bray, 2009; Gilbert & Musu, 2008).
Learning Centres

Centres can take many forms and provide students with opportunities to enrich or reinforce learning without being repetitive. Centres might be permanent fixtures in your classroom, such as the water table or blocks area; they could be a revolving set of tasks used repeatedly over the year that might involve familiar activities (e.g., tangram puzzles) or games (e.g., Race to a Loonie); or they may focus on a few different activities to introduce a new concept informally, extend activities taken up as a whole class, or practise previously learned skills. Centres may be located throughout the room, requiring students to move from one centre to another, or they might be bags or bins that are taken to students’ desks or tables. In each case, centres need to include the materials needed along with task cards with clear directions and expectations (see Figure 6.1).

Thumb to Pinkie

Grade 3 Target: Estimate, compare, and measure length in centimetres.
Materials: centimetre ruler, string, paper

Stretch your fingers out wide.
Imagine a straight line from your thumb to your pinkie.
Now imagine the line that is not straight tracing around your fingers.

Estimate: Which line is longer? Estimate the length of the straight line. Record your estimate.
Measure: How will you measure the lines? Measure and record the lengths of each line.
Compare: Which line was longer? Did you think it would be longer? Write about what you found.

FIGURE 6.1 Task card example.

Think-Pair-Share

Think-pair-share (Lyman, 1981) or think-write-pair-share is a strategy that is used across the curriculum to provide independent think time to students and enhance the quality and depth of their responses. While there are variations of this strategy, in general the teacher poses a question to the class and asks each student to think about it quietly for a few moments (e.g., 30 seconds). They then turn to a partner and discuss the question for a couple of minutes. Afterwards, volunteers are asked to share their ideas with the class. Think-pair-share is useful for simply considering how to approach a challenging task, or for sharing specific solutions, as the following example illustrates:

Ask: “How would you solve the following question mentally: 347 – 159?”
Think: “Take a few moments to think about it. Put your thumb up when you think you have an answer.”
Pair: “Turn to your partner. Do you both have the same answer? Did you solve it the same way or in different ways?”
Share: “Okay, Elliot and Zach. You two were discussing an interesting strategy. Could you share that with the class?”
“We each did it a different way, but when we were talking, we figured out an even better way. We said 350 – 150 is 200. But then you have to subtract 12 to keep the difference the same so that’s just 188.”

The sharing of the different mental strategies could continue until no new strategies were shared. Think-pair-share allows students time to formulate their own ideas and become confident in communicating them, but pairing allows students to clarify, collaborate, and expand on each other’s ideas as well.

Choice Boards, Menus, and Think-Tac-Toe

Choice boards, Menus (Burns 2007; Tomlinson, 2003), and Think-Tac-Toe (Tomlinson, 2003) are all differentiation strategies that offer choices to students. Offering choices must be done strategically because regardless of the choices made, students need to be engaged in interesting tasks that address learning expectations and challenge their mathematical knowing.

With choice boards and menus, teachers provide three or more tasks based on the current unit of study. The teacher specifies which task must be completed (the main dish), selects a specified number of other tasks (appetizers and side dishes), and provides some optional but irresistible tasks (desserts) (Wormeli, 2006). Think-Tac-Toe is a set of nine tasks placed in a 3 × 3 grid. The expectation is for students to complete three tasks where one must be the middle square (the mandatory task) and the other two choices complete a horizontal, vertical, or diagonal line (See Figure 6.2).

Graphic Organizers

Writing can also be used to help students connect representations. A common graphic organizer is the four-box table. In each box, students record the problem, an explanation, an illustration, and the general math concept (Wu, An, King, Ramirez, & Evans, 2009; Zollman, 2009).

The requirements for each box can be adapted, as needed, for the content area; for example, the Frayer Model (Frayer, Federick, & Klausmeier, 1969), commonly used in
language learning, can be adapted to develop vocabulary and understanding in geometry:

Many graphic organizers can be used to elicit writing in mathematics (and math journals): They help students know what to write. In the case of the one pictured here, you don’t even need a handout; just fold a piece of paper in fourths and then dog-ear the inside corner on the fold. When you unfold, you will have a paper divided as shown.

Diversity in Our Classrooms

The differentiated strategies described above are just a few of the common ones you will find useful in the teaching of mathematics for all students. Throughout the remainder of this chapter, we will continue to build your cache of differentiation strategies, indicating which are especially effective with specific populations of students, including Aboriginal students, students who are culturally and linguistically diverse, and students with special learning needs.

First Nations, Métis, and Inuit Students

Enrollment and completion rates for First Nations, Métis, and Inuit (FNMI) students in Grade 12 and post-secondary education programs have been improving in recent years, but remain far lower than for non-Aboriginal students. FNMI students are over-represented in special education classes and under-represented in higher-level mathematics courses in high school and post-secondary programs. Canada’s Aboriginal population is growing more rapidly than other population segments and increased by over 20 percent after the turn of the century, thereby making up a growing proportion of the student population in elementary and middle years schooling. Strategies for working with FNMI students are as diverse as they would be in any student population, but the literature provides some recommendations to help teachers of mathematics become more cognizant of the potential opportunities for supporting the learning of Aboriginal students in their classrooms.

Ways of Learning. There are over 60 Aboriginal languages spoken in Canada including Cree, Ojibway, Mi’kmaq, and Inuktitut, to name just a few. In general, Aboriginal cultures tend to have some similarities in language, world view, and hence ways of learning, that may distinguish them from Western perspectives. Indigenous languages tend to originate from an oral tradition: This indicates ways of learning that emphasize hands-on experiences, apprenticeship perspectives, emphasis on mastery of skills, and visual-spatial learning (Wagner & Lunney Borden, 2011).
Learning, then, is viewed as holistic, experiential, and rooted in relationships, nature, movement, tradition, language, and culture (Cappon, 2008). Listening, watching, and doing mathematics through relationship and real-life modelling are important ways of learning.

**Verb-Based Languages.** Little Bear (2000) explained that “Language embodies the way a society thinks,” and “Aboriginal languages are, for the most part, verb-rich languages that are process- or action-oriented” (p. 78). Lunney Borden (2011; 2013) provides a useful illustration of the differences not only in language, but in ways of thinking influenced by language when working with a teacher and Grade 3 Mi’kmaq students in Nova Scotia. Geometry is a mathematical domain that relies on specific vocabulary for objects and attributes—or geometric nouns. When describing the face of a cube, the teacher emphasizes movement when she says, “I use my face to look at you and the cube can look at you with all six of his faces.” An edge is less of an attribute and more of an action when she prompts students to think, “we go over the edge” as they move from one face to another. A pyramid is something that cannot stand on its head, but prisms can. Borden suggests seeking out not only the translated word in an Indigenous language, but the meaning and grammatical structures that draw on the use of verbs.

**Culturally Responsive Teaching Strategies and Materials.** Culturally responsive FNMI education, as described and developed by teachers in a study with Nicol, Archibald, and Baker (2013), emphasizes several key aspects that draw on the multitude of Aboriginal ways of learning and language-based perspectives of the world that include attention to place, story, relationship, and hands-on, inquiry-based learning.

Working collaboratively with families, elders, and community members is essential in teaching and learning; otherwise, as Doolittle (2006) cautions, cultural objects are treated as add-ons. Without community support, approaches to teaching may lead to an oversimplification whereby a tipi is used as an example of a cone, and baskets and beadwork are completed as simple patterning activities without recognition of their cultural connections.

Given the diversity within First Nations, Métis, and Inuit cultures, prescribing particular approaches risks overgeneralizing strategies for all FNMI students. Building local relationships is vital for creating relevant opportunities within educational and community contexts.

**Students Who Are Culturally and Linguistically Diverse**

Canadian classrooms are increasingly diverse. Over 20 percent of Canada’s current population comprises immigrants from all over the world (Statistics Canada, 2011). Overall, the majority of immigrants are from the United Kingdom, but recent trends show that the greatest percentage of immigrants now entering the country are from China, India, and the Philippines. Most of the children from these families live in households where the main language spoken is something other than English or French.

You have probably heard that “mathematics is a universal language.” This common misconception can lead to inequities in the classroom. Conceptual knowledge (e.g., what subtraction means) is universal. Procedures (e.g., how you subtract) and symbols are culturally determined, and are not universal. For example, as you will read in Chapters 12 and 13, there are many algorithms for whole-number operations. Compare the following two subtraction algorithms:

\[
\begin{array}{c}
\text{783} \\
\text{– 47} \\
\hline
\text{36}
\end{array}
\quad \begin{array}{c}
\text{83} \\
\text{– 47} \\
\hline
\text{36}
\end{array}
\]

Can you follow what the first student did? If you learned subtraction in Canada, this likely looks familiar. It is called the “decomposition” algorithm for subtraction. But, if you learned subtraction in a European country, you may have learned the second algorithm, called “equal additions” (which is a hint as to the procedure used) or the “Austrian method.” Can you follow the second example? It uses the meaning of subtraction as “difference” to add equal quantities to both numbers. (Incidentally, children who learn this method of subtraction are often able to do subtraction mentally rather than continuing to rely on paper-and-pencil procedures.) The critical issue, though, is less about whether you can follow an alternative approach, and more about how you will respond when you encounter a student using such an approach. Will you require the student to use a specific procedure, disregarding the way they had learned it? Or will you ask the student to share with other students how they did it and why it works? By doing the latter, you can show that you value the student’s culture.

**Culturally Diverse Students**

Culturally relevant mathematics instruction is not just for recent immigrants; it is for all students, including students from different ethnic groups, socioeconomic statuses, and so on. Culturally relevant mathematics instruction takes into consideration content, relationships, cultural knowledge, flexible approaches, accessible learning contexts, a responsive learning community, and crosscultural partnerships (Averill, Anderson, Easton, Te Maro, Smith, & Hynds, 2009). It is complex. A learning strategy may be highly effective in one setting, and yet not work in a different setting. Each of the following four overlapping approaches offers ways to make your mathematics teaching culturally relevant.
Focus on Important Mathematics. Too often, our first attempt to help students is to simplify the mathematics, which just lowers the chance of students learning the content. For students who struggle with reading (including but not only ELLs), a common modification is to remove the language from the lesson. This has the effect of reducing the mathematics to skill development, which is rarely connected to real experiences. Culturally relevant instruction stays focused on the big ideas of mathematics and helps students engage in and stay focused on the big ideas. Engage students in productive struggle and in making connections between mathematics concepts.

Make the Content Relevant. There are really two components for making content relevant. One is to think about the mathematics: “Is the topic connected meaningfully to other content?” This is really important in teaching for all students, as some students in the classroom may not have learned a skill they need. Rather than assign such a student to a remedial lesson, you should instead infuse the related prior knowledge. For example, begin a lesson on finding the centre of a circle by asking students to draw the diameter to locate the centre. Question students as they work: “How do you know if a line is a diameter? Do all diameters have the same measure? How many diameters do you need in order to establish the centre of the circle?” By incorporating the prior knowledge of diameters as they relate to circles and engaging students in explorations, you make the content relevant.

Second, making content relevant is about contexts. What contexts can bring meaning to the mathematics? There are many! Historical or cultural topics abound. Students can become personally engaged in mathematics by examining their culture’s impact on the ways they use, practice, and think about mathematics. A study of mathematics within other cultures provides opportunities for students to “put faces” on mathematical contributions. For example, when focusing on counting and place value, look at the Mayan place value system to think more deeply about reasoning about the structure of number (Farmer & Powers, 2005).

Six mathematical behaviours have been found across time and culture: counting, measuring, locating, designing and building, playing games (e.g., “Mancala”), and explaining (e.g., telling stories) (Bishop, 2001). When your curriculum takes you to one of these topics, invite students and their families to share their experiences and use these experiences to engage in the content.

Incorporate Students’ Identities. This consideration overlaps with that of relevant content, but is worth its own discussion. Students must see themselves in mathematics and see that mathematics is a part of their culture. You don’t need to be a historian to build cultural connections—just ask your students. In a project focused on helping teachers recognize students’ identities, teachers asked students to create a poster to show outsiders how many students were in their classroom. Many representations created by the K–2 students included students’ skin colour, hair colour, and gender. These traits became part of bar graphs and sorting activities with the students’ identities at the centre of the learning (McCulloch, Marshall, & DeCuir-Gunby, 2009).

Both researchers and teachers have found that telling stories about mathematics in their own lives, or asking students to tell math stories, makes the mathematics relevant to students and can raise student achievement (Turner, Celedón-Pattichis, Marshall, & Tennison, 2009). For example, you can ask students to bring a math story from a family trip to the grocery store (Butterworth & Lo Cicero, 2001). Or students can develop math story problems from photos. These photos could be pictures cut out from their community newspaper, family photographs they’ve brought from home, or their own pictures taken using digital cameras (Lemons-Smith, 2009; Leonard & Guha, 2002). Similarly, students can explore an artifact from their culture, or one that has captured their interest, presenting the mathematics of the artifact (e.g., a game or measuring device) (Neel, 2005).

The following is a teacher’s story of how she incorporated family history and culture into her class by reading “The Hundred Penny Box” (Mathis, 1986). The story describes a 100-year-old woman who remembers one year’s important event in her life for every one of her hundred pennies. Each penny is more than a piece of money; it is a “memory trigger” for her life.

Taking a cue from the book, I asked each child to collect one penny from each year they were alive starting from the year of their birth and not missing a year. Students were encouraged to bring in additional pennies their classmates might need. Then the students consulted with family members to create a penny timeline of important events in their lives. Using information gathered at home they started with the year they were born listing their birthday and went on to record first steps, accidents, vacations, pets, and births of siblings in those early years. Then students determined how many years between certain events or calculated their age when they adopted a pet or learned to ride a bicycle. These events were to be used in the weeks and months to come as subjects of story problems and other mathematics investigations.

Students need opportunities to engage in discussion about family and culture, but keep in mind that typical discourse patterns in classrooms may not feel natural to all students. Inviting students to share and providing explicit guidance in how to participate can increase the participation rates of all students.

Ensure Shared Power. You determine who has the authority in your classroom and who listens to whom. In too many classrooms, the teacher has the power—telling students
whether answers are right or wrong (rather than having students determine correctness through reasoning), telling students exactly how to solve problems (rather than giving choices for how they will engage in the problem), and determining who will solve which problems (rather than allowing flexibility and choice). Instead, establish a classroom environment where everyone feels their ideas are worth consideration. The way that you assign groups, seat students, and call on students sends clear messages about who has power in the classroom. Distributing power among students leads to empowered students.

Each day’s lesson provides new opportunities and challenges as you think about how you will make lessons culturally relevant. If you focus on important mathematics, make content relevant, incorporate students’ identities, and ensure shared power as part of what you naturally think about as you plan, teach, and assess, then you are likely going to lead a classroom where all students are challenged and supported.

**Students Who Are English Language Learners (ELLs)**

English language learners enter the mathematics classroom from homes in which English is not the primary language of communication. Although a person might develop conversational English language skills in a few years, it takes as many as seven years to learn “academic language,” which is the language specific to a content area such as mathematics (Cummins, 1994). Academic language is harder to learn because it is not used in a student’s everyday world. When learning about mathematics, students might be learning content in English that they have no words for in their native language. For example, in studying the measures of central tendency (mean, median, and mode), they may not know words for these terms in their first language, increasing the challenge for learning academic language in their second language. In addition, story problems are difficult for ELLs not just due to the language but also to the fact that sentences in story problems are often structured differently from sentences in conversational English (Janzen, 2008).

Teachers of English to Speakers of Other Languages (TESOL) argue that ELLs need to use English (and their native language) to read, write, listen, and speak as they learn appropriate content. Creating effective learning for ELLs involves integrating principles of bilingual education with inquiry-based mathematics teaching. Among the many classroom supports for students who are learning English, the strategies discussed in this section are the ones that appear in the research literature most frequently as critical to increasing the academic achievement of ELLs in mathematics classrooms.

Rather than assume that mathematics is a universal language or limit the use of mathematical terms and symbols, teachers need to maximize the language used, but do so in multiple ways to support language development while keeping expectations for mathematics learning high. In the following example, the teacher uses several techniques that provide support for her ELL learners.

Ms. Nygard is working on a Grade 4 geometry lesson that requires students to apply their knowledge of the geometric properties of three-dimensional shapes (polyhedra) to identify prisms and pyramids by their distinguishing features (e.g., the number of and shape of their faces). The task requires students to examine a set of face cards for a three-dimensional shape, decide which shape it is, and then check whether they have selected the correct three-dimensional shape by examining an actual model. Ms. Nygard has a student from Ethiopia who has been in Canada for only eight months and knows very little English, and a student from Sri Lanka who has been here for about two years. These two students may not be familiar with the terms “prism” and “pyramid” and will have difficulty identifying which set of face cards represents a prism and which one represents a pyramid. The student from Ethiopia may also be confused by the word “face,” which in common English usage has a different meaning.

To ensure that these students (as well as the rest of the class) are familiar with the words “prism” and “pyramid,” Ms. Nygard addresses their meaning before launching into the lesson. She writes the words “prism” and “pyramid” on the board and asks students what they are. She allows time for students to discuss with a partner, then to share with the whole class. Ms. Nygard makes sure that she has different prisms and pyramids displayed at the front of the room. Students also discuss the use of the word “face” so that her English language learners appreciate its use in the context of geometry, as well as in everyday life. Upon completion of the discussions, Ms. Nygard explains to the class that today they are going to apply their knowledge of the properties of three-dimensional shapes to identify prisms and pyramids. Ms. Nygard models how the task is to be carried out. First, she lays out a set of cards for the class and asks them, “What three-dimensional shape do you think this is? Why do you think so?” She has students discuss with a partner. She makes certain that the two English language learners are paired with a partner who can offer necessary language support. Then, a student is invited to give an answer and an explanation for her or his choice. Prism and pyramid models can be used to check the student’s response.

**Pause AND REFLECT**

Review Ms. Nygard’s lesson. What specific strategies to support students who are learning English (ELLs) can you identify?

Discussion of the words “prism,” “pyramid,” and “face” using a think–pair–share technique recognized the potential language confusion. It allowed students the chance to
talk about the terms before becoming perplexed by the task. Using visuals and concrete models (the cards and the 3-D shapes) also provided support, so the ELL students could succeed in this task. As well, Ms. Nygard had ELL students work with a partner who could offer language support as they carried out the task. Most importantly, Ms. Nygard did not diminish the challenge of the task with these strategies. If she had altered the task, for example, by using simpler shapes, she would have lowered her expectations. Conversely, if she had simply posed the problem without taking time to discuss the meanings of the words, provide visuals, and model the task, she would have kept her expectations high but failed to provide the support that would enable her students to succeed. Instead, she took the necessary steps to ensure that all students would be successful.

Honour the Use of Native Languages. One out of six people in Canada speak a language other than English or French at home. Recent immigration is changing the language landscape. Mandarin, Philippines-based Tagalog, Arabic, and Hindi are among the fastest rising languages (Statistics Canada, 2011). Research strongly supports the use of a student’s native language in the classroom (Haas & Gort, 2009; Moschkovich, 2009; Setati, 2005). Valuing a student’s language is one of the ways you value their cultural heritage. In a mathematics classroom, students can experience greater success if they are able to communicate in their native language while continuing their English language development. For example, a good strategy for students working in small groups is to have students discuss the problem in their preferred language. If a student knows enough English, then the presentation in the after phase can be shared in English. If the student knows little or no English, then he or she can explain in Mandarin using a translator. Bilingual students will often code-switch, moving between two languages. Research indicates that this practice of code-switching supports mathematical reasoning because the student is selecting the language from which they can best express their ideas (Moschkovich, 2009).

Write and State Content and Language Objectives. Every lesson should begin with telling students what they will be learning. This doesn’t mean telling students what to do and how exactly to do it, but instead, stating the larger purpose and providing a map of the boundaries. If students know the purpose of the lesson, they are better able to make sense of the details when challenged by some of the oral or written explanations. By explicitly including language expectations, students know language they will be developing alongside the mathematical goals. For example,

Today you will:
1. Analyze properties and attributes of three-dimensional solids (mathematics).

2. Describe in writing and orally a similarity and a difference between two different solids (language and mathematics).

Build Mathematical Background. Building background takes into consideration native language and culture, as well as content. If possible, use a context and appropriate visuals to help students understand the task you want them to solve. For example, Pugalee, Harbaugh, and Quach (2009) spray-painted a coordinate axis in the field so that students could build background related to linear equations. Students were given various equations and contexts and had to physically find (and walk to) a point on the giant axis, creating human graphs of lines. This nonthreatening, engaging activity helped students make connections between what they had learned and what they needed to learn.

Some aspects of English and mathematics are particularly challenging to ELLs (Whiteford, 2009/2010). For example, the names and symbols of teen numbers in English don’t directly correspond to place value. (In other languages, the teens typically follow the same pattern of the other decades.) The Cantonese characters, sounds, and meanings, however, do represent place value more directly.

<table>
<thead>
<tr>
<th>Number</th>
<th>Character</th>
<th>Reading</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>—</td>
<td>yat</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>—</td>
<td>yi</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>—</td>
<td>saam</td>
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<td>4</td>
<td>—</td>
<td>sei</td>
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<td>10</td>
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<td>sap</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>—</td>
<td>sap yat</td>
<td>10 + 1</td>
</tr>
<tr>
<td>12</td>
<td>—</td>
<td>sap yi</td>
<td>10 + 2</td>
</tr>
<tr>
<td>20</td>
<td>—</td>
<td>yi sap</td>
<td>2 x 10</td>
</tr>
<tr>
<td>21</td>
<td>—</td>
<td>yi sap yat</td>
<td>2 x 10 + 1</td>
</tr>
<tr>
<td>22</td>
<td>—</td>
<td>yi sap yi</td>
<td>2 x 10 + 2</td>
</tr>
</tbody>
</table>

Source: Based on figure by Shinji Takasugi.

In English, the teen numbers sound a lot like their decade number. For example, fifteen and fifty, and sixteen and sixty sound very similar. Emphasizing the ‘n’ in “teen” helps ELLs hear the difference.

When you encounter these situations, and others, additional time is needed to build background and draw attention to how you recognize the intended meaning of the words.
Use Comprehensible Input. Using comprehensible input means that the message you are communicating is understandable to students. Modifications include simplifying sentence structures and limiting the use of nonessential or confusing vocabulary. Note that these modifications do not lower expectations for the lesson. Sometimes, teachers put many unnecessary words and phrases into questions, making them less clear to English language learners. Compare the following sets of teachers’ directions:

Not Modified: “You have a lab sheet in front of you that I just gave out. For every situation, I want you to determine the total area for the shapes. You will be working with your partner, but each of you needs to record your answers on your own paper and explain how you got your answer. If you get stuck on a problem, raise your hand.”

Modified: “Please look at your paper. (Holds paper and points to the first picture.) You will find the area. What does area mean? (Allows wait time.) How can you calculate area? Talk to your partner. (Points to mouth and then to a pair of students as she says this.) Write your answers. (Makes a writing motion over paper.)”

Notice that three things have been done: Sentences have been shortened, confusing words have been removed, and related gestures and motions have been added. Also notice the wait time the teacher gives. It is very important to provide extra time after posing a question or giving instructions to allow ELLs time to translate, make sense of the request, and then participate.

Explicitly Teach Vocabulary. Intentional vocabulary instruction is not only relevant to English language learners, but must also be part of mathematics instruction for all students. Consider the following strategies:

- Use math word walls with visuals (see Figure 6.3), list key vocabulary in a prominent location, and review vocabulary needed for the task.
- Use graphic organizers, anchor charts, recording tables, and concept maps (see Figure 6.4).

- Play games focused on vocabulary development (e.g., charades, “$10,000 Pyramid,” “Concentration”).
- Encourage students to make their own vocabulary cards or personal dictionaries that include the word in English, translations in their mother tongue, pictures, and a student-made description (not a formal definition) (Kersaint, Thompson, & Petkova, 2009).
- Analyze the task for language pitfalls and identify words that need to be discussed.

All students benefit from an increased focus on language; however, too much emphasis on language can diminish the focus on the mathematics. Importantly, the language support should be connected to the selected task or activity. As you analyze a lesson, you must identify terms related to the mathematics and to the context that may need explicit attention. Consider the following task (National Center for Education Statistics, 2009):

Sam did the following problems:

| 2 + 1 = 3  
| 6 + 1 = 7 |

Sam concluded that when he adds 1 to any whole number, his answer will always be odd.

Is Sam correct? ______________

Explain your answer.

In order for students to engage in this task, the terms “even” and “odd” must be understood. Both terms may be known for other meanings (“even” can mean “level” and “odd” can mean “strange”). “Concluded” is not a math word, but must be understood if the student is to understand the meaning of the problem. In this case, rather than build meaning for this word, a new word could be used that is more familiar, like, “Sam has an idea that….” Finally, you must give guidance on how students will explain—must it be in words, or are pictures or diagrams acceptable?

“Odd” and “even” are among hundreds of words that take on different meanings in mathematics from everyday activities. Others include “product,” “mean,” “sum,” “factor,” “acute,” “division,” “difference,” “similar,” and “angle.” Also, when planning, when using text-based student materials, and when teaching, consider the following:

- Watch for homonyms (e.g., base, even, mean, similar, product) and homophones (e.g., one/one, two/too/to, eight/ate, half/have, cent/scent, weight/wait, whole/whole, root/route).
- Provide tasks in written format with visuals, use real objects related to the selected task, and provide oral instructions.
- Use real objects and multimedia such as virtual manipulatives, applets, and dynamic geometry tools (e.g., Geometer’s Sketchpad) to help illustrate the concepts you are using.
Maximize Opportunities for Discourse. ELLs need opportunities to speak, write, talk, and listen in nonthreatening situations. Because ELLs cannot always explain their ideas fully without rehearsal, rather than just calling on someone else, pressing for details is important. This pressing is not just so the teacher can decide whether the idea makes sense; it is so that other students can make sense of the idea (Maldonado, Turner, Dominguez, & Empson, 2009).

Since use of language is extra important, opportunities for students to practise phrases or words through choral response or through pair-share are needed. Place pairs of students that have the same first language together with another pair of English speakers (Garrison, 1997; Khisty, 1997). All students need opportunities to explain and defend their solutions, so allowing students to work out their explanation in a small group before sharing with a full class will help build language skills and confidence.

Prevention Models and Interventions for All Students

A process for achieving higher levels of performance for all students includes an approach called “response to intervention (RtI).” This is a prevention model that emphasizes ways for struggling students to get immediate assistance and support, rather than waiting for students to fail before they receive assistance. Prevention models are centred on three interwoven elements: high-quality curriculum, instructional support (interventions), and formative assessments that capture students’ strengths and weaknesses. Prevention models were designed to determine whether low achievement was due to a lack of high-quality mathematics
(i.e., “teacher-disabled students”) (Baroody, 2011; Ysseldyke, 2002) or due to an actual learning disability.

Response to Intervention. RTI is a tiered student support system that focuses on the results of implementing instructional interventions in a model of prevention. Each tier in the triangle represents a level of intervention with corresponding monitoring of results and outcomes, as shown in Figure 6.5. The foundational and largest portion of the triangle (tier 1) represents the instruction that should be used with all students—differentiated instruction and universal screening of all students. At tier 1, a balanced set of different assessments should be used to monitor progress and allow all students to demonstrate the knowledge and skills expected by grade-level standards.

Tier 2 represents students who did not reach learning expectations during tier 1 activities but are not yet considered as needing special education services. Students in tier 2 should receive additional targeted instruction (interventions) using more explicit instruction with systematic teaching of critical skills, more intensive and frequent instructional opportunities, and more supportive and precise prompts to students (Torgesen, 2002). If further assessment (such as diagnostic interview results) reveals favourable progress, students move back to tier 1.

If challenges and struggles still exist, the interventions can be adjusted or, in rare cases, the students are referred to the next tier of support. Tier 3 is for students who need more intensive levels of assistance, which may include comprehensive mathematics instruction or a referral for special education evaluation or special education services. Strategies for the three tiers are outlined in Table 6.1.

Research into the use of prevention models such as RtI reveal that although most students remain in tier 1, approximately 15 percent of students fail to demonstrate the full growth expected and are moved to tier 2 for more intense instructional methods (Fuchs & Fuchs, 2001). Eventually, nearly 40 percent of students who move to tier 2 respond to the interventions and return to tier 1. Only about 13 percent of the original group that moved to the second tier is considered for individual services—usually from a special educator—at the tier 3 level (Fuchs & Fuchs, 2005, 2007).

Progress Monitoring. A key to the prevention model is the monitoring of students’ progress. The data from these formative assessments are what guide the movement within tiers. Teachers can collect evidence of student understanding of concepts through the use of diagnostic interviews. Another approach is to assess students’ growth toward fluency in basic facts, an area that is well documented as a barrier for students with learning disabilities (Mazzocco, Devlin, & McKenney, 2008). According to Woodward (2006), integrating strategy instruction in computation (see Chapter 10), along with frequent timed practice, helps students develop automaticity of number facts and improve number sense, including estimation and mental math skills.
The collection of information gathered from short frequent assessments will reveal whether students are benefitting as expected or if more intensive instructional approaches need to be put into practice.

**Students with Mild Disabilities**

Students with learning disabilities have very specific difficulties with perceptual or cognitive processing and may be identified as needing tier 3 services. These difficulties may affect memory; general strategy use; attention; the ability to speak or express ideas in writing; the ability to perceive auditory, visual, or written information; or the ability to integrate abstract ideas. Although each student will have a unique profile of strengths and weaknesses, there are ways to support students with mild disabilities in all phases of mathematics lesson planning, teaching, and assessment.

Research-based strategies for teaching students with difficulties in mathematics (such as students needing interventions in tier 2 or tier 3 of a prevention model, such as RtI) include systematic and explicit strategy instruction, think-alouds, concrete and visual representations of problems, peer-assisted learning activities, and the provision of formative assessment data to students (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). These proven approaches are, in some cases, quite different from those at tier 1. The strategies described here are interventions for use with the small subset of students for whom the initial interventions were ineffective.

**Explicit Strategy Instruction.** Explicit instruction is often characterized by highly structured, teacher-led instruction on a specific strategy. The teacher does not merely model the strategy and have students practise it, but
attempts to illuminate the decision making that may be troublesome for these learners. In this model, the teaching routines used include a tightly scripted sequence that moves from modelling, to prompting students through the model, to practice. Instruction is highly organized in a step-by-step format and involves teacher-led explanations of concepts and strategies, including the critical connection building and meaning making that help learners relate new knowledge with concepts they know. Let’s look at an example of a classroom teacher using explicit instruction:

As you enter Mr. Logan’s classroom, you see a small group of students seated at a table listening to the teacher’s detailed explanation and watching his demonstration of equivalent fraction concepts. The students are using manipulatives, as prescribed by Mr. Logan, and moving through carefully selected tasks. He tells the students to take out the red “one-fourth” pieces and asks them to check how many “one-fourths” will exactly cover the blue “one-half” piece. As he begins, Mr. Logan often asks, “Is that a word you know?” Then, to make sure they don’t allow for any gaps or overlaps in the pieces, he asks them to talk about their process with the question, “What are some things you need to keep in mind as you place the fourths on the half?” Mr. Logan writes their responses on the adjacent board as \( \frac{2}{4} = \frac{1}{2} \). Then he asks them to compare the brown “eighths” and the yellow “sixths” to the piece representing one-half and records their responses. The students are taking turns answering these questions out loud. During the lesson Mr. Logan frequently stops the group, interjects points of clarification, and directly highlights critical components of the task. For example, he asks, “Are you surprised that it takes more eighths to cover the half than fourths?” Vocabulary words, such as “numerator” and “denominator” are written on the math wall nearby, and the definitions of these terms are reviewed and reinforced throughout the lesson. At the completion of the lesson, students are given several similar examples of the kind of comparisons discussed in the lesson as independent practice.

A number of aspects of explicit instruction can be seen in Mr. Logan’s approach to teaching fraction concepts. He employs a teacher-directed teaching format, prescribes the use of manipulatives, and incorporates a model-prompt-practice sequence. This sequence starts with verbal instructions and demonstrations with concrete models, followed by prompting, questioning, and then independent practice. The students are deriving mathematical knowledge from the teacher’s oral, written, and visual clues.

As students solve problems, they are also given explicit strategy instruction to guide them in carrying out tasks. They are asked to read and restate the problem, draw a picture, develop a plan by identifying the type of problem, write the problem in a mathematical sentence, break the problem into smaller pieces, carry out operations, and check their answers using a calculator. These prompts structure the entire learning process from beginning to end.

Unlike in inquiry-based instruction, the teacher models and explains each step using terminology that is easily understood by students with disabilities who did not discover them independently through earlier tier 1 or 2 activities. Yet, consistent with what we know about how all students learn, students are still engaged in doing mathematics (not just skill development).

Concrete models can support very different teaching approaches. For example, a teacher who is using explicit strategy instruction to demonstrate a multiplication array with cubes might say, “Watch me. Now make a rectangle with the cubes that looks just like mine.” In contrast, a teacher with a more inquiry-oriented approach might say, “Using these cubes, how can you show me a representation for \(4 \times 5\)?” While more structured, the use of concrete models provides access to abstract concepts.

There are a number of possible advantages to the use of explicit strategy instruction for students with disabilities. This approach helps uncover or make overt the covert thinking strategies that support mathematical problem solving. Students with disabilities may otherwise not have access to these strategies. Explicit approaches are also less dependent on the student to draw ideas from his/her past experience or to operate in a self-directed manner.

Explicit strategy instruction can also have disadvantages for students with disabilities. Some aspects of this approach rely on memory, which can be one of the weakest areas for some students with special needs. Taking a known weakness and building a learning strategy around it is not productive. There is also the concern that highly teacher-controlled approaches promote prolonged dependency on teacher assistance. This is of particular concern for students with disabilities because many of them are described as “passive learners.” Students learn only what they have the opportunity to practise. Students who are never given opportunities to engage in self-directed learning (based on the assumption that this is not an area of strength) will be deprived of the opportunity to develop skills in this area. In fact, explicit instruction is intended to move from a highly structured, single-strategy approach to multiple models, including examples and non-examples. It also includes immediate error correction with the fading of prompts to help students move towards independence.

Another possible challenge of explicit approaches is the depth of understanding that can be expected when students do not have opportunities to actively engage in the doing of mathematics and hence, their ability to retain, generalize, and apply information—all skills that are vital to long-term success in mathematics. Explicit instruction, to be effective, must include making mathematical relationships explicit (so that students don’t just learn how to do that day’s mathematics, but also make connections to other mathematical ideas). Since this is one of the major research findings regarding how students learn, it must be central to the learning strategies used with students with mild disabilities.
Consider a problem in which Grade 4 students are given the task of determining how much paint will be needed to cover the walls of their classroom. Rather than merely demonstrating, for example, how to use a ruler to measure the distance across a wall, the think-aloud strategy would involve the teacher talking through the steps and identifying the reasons for each step while measuring the space. As the teacher places a mark on the wall to indicate where the ruler ended in the first measurement, she states, “I used this line to mark off where the ruler ends. How should I use this line as I measure the next section of the wall? I know I have to move the ruler, but should I copy what I did the first time?” All of this dialogue occurs prior to placing the ruler for a second measurement. Often teachers share alternatives about how else they could have carried out the task. When using this metacognitive strategy, teachers try to talk about and model possible approaches in an effort to make their invisible thinking processes visible to students.

Although you will choose strategies as needed, your goal is always to work toward high student responsibility for learning. Movement to more connected forms of understanding content is a primary goal. For some, formal support along the way is necessary (explicit strategy instruction); for others, encouragement will work (peer-assisted learning). Other students can be successful on their own with visual representations (CRA/CSA approach). All people can relate to the need to have different levels of support during different times of their lives or under different circumstances, and it is no different for students with special needs (see Table 6.2).

### Students with Moderate/Severe Disabilities

Students with moderate/severe disabilities often need extensive modifications and individualized supports to understand the mathematics curriculum. This population of students may include those with severe autism, sensory disorders, limitations affecting movement, processing disorders (e.g., intellectual disabilities), cerebral palsy, or combinations of multiple disabilities stemming from specific issues, such as fetal alcohol spectrum disorder or otherwise.

Originally, the curriculum for students with severe disabilities was called “functional,” in that it often focused on life-related skills such as managing money, telling time, using a calculator, measuring, and matching numbers to complete such tasks as entering a telephone number or identifying a house number. Now the emphasis is on numeracy through real-world representations as a way to prepare all students to be mathematically literate citizens. Using money to study place-value concepts or posing problems in the context of making purchases are approaches with multiple benefits for students with severe disabilities.
At a beginning level, students work on identifying quantities and numbers by holding up fingers or pictures. To develop number sense, counting up can be linked to counting daily tasks to be accomplished, and counting down can mark a period of cleanup after an activity or to complete self-care routines (brushing teeth). Students with moderate or severe disabilities should have opportunities to use measuring tools, compare graphs, explore place-value concepts (often linked to money use), use the number line, and compare quantities. When possible, the content should be connected to life skills (e.g., shopping, preparing food) and possible features of jobs—such as restocking supplies (Hughes & Rusch, 1989). At other times, just linking mathematical learning objectives to everyday events is practical. For example, when studying the operation of division, figuring how candy can be equally shared at Halloween or counting number after 5 are shared by other students during the after portion of the lesson. Forgets how to start the problem-solving process.

Do not believe that all basic facts must be mastered before students with moderate or severe disabilities can move forward in the curriculum; students can learn geometric or measuring concepts without having mastered addition and subtraction facts. Geometry, for students with moderate and severe disabilities, is more than merely identifying shapes, but is in fact critical for helping them to orient themselves in the real world. Using maps related to transit routes as teaching materials can support students' use of public transportation and is also an opportunity to discuss geometric concepts, such as parallel and perpendicular lines. Students who learn to count bus stops and judge time will navigate their world more successfully.

Table 6.3 offers ideas across the curriculum appropriate for teaching students with moderate to severe disabilities. When possible, blend the mathematics curriculum with the basic skills a student needs in a practical living context. If other students study the measures of various angles of triangles, the student with moderate disabilities can match right-angled triangles to a model on a mat as part of learning about right angles. In this example, the content area remains within grade-level mathematics objectives while being adapted to meet the needs of students with moderate disabilities to grow in concepts, vocabulary, and symbol use.

The following lists comprise additional task and teaching strategies for supporting students with moderate and severe disabilities:

Task Considerations

- **In vivo.** Use real-life (in vivo) applications so students can see how mathematics concepts are useful in everyday activities.
- **Simplify visual displays.** Design tasks and assessments with more white space. Increase font size and reduce

TABLE 6.2

<table>
<thead>
<tr>
<th>Stumbling Blocks</th>
<th>What Will I Notice?</th>
<th>What Should I Do?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student has trouble forming mental representations of mathematical concepts</td>
<td>• Can’t interpret a number line</td>
<td>• Explicitly teach the representation—for example, exactly how to draw a diagram.</td>
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<tr>
<td></td>
<td>• Has difficulty going from a story about a garden plot (to set up a problem on finding area) to a graph or dot paper</td>
<td>• Use larger versions of the representation (e.g., number line) so that students can move to or interact with the model.</td>
</tr>
<tr>
<td>Student has difficulty accessing numerical meanings from symbols (issues with number sense)</td>
<td>• Has difficulty with basic facts; for example, doesn’t recognize that 3 + 5 is the same as 5 + 3, or that 5 + 1 is the same as the next counting number after 5</td>
<td>• Explicitly teach multiple ways of representing a number showing the variations at the exact same time.</td>
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<tr>
<td></td>
<td>• Use multiple representations for a single problem to show it in a variety of ways (blocks, illustrations, and numbers) rather than using multiple problems.</td>
<td></td>
</tr>
<tr>
<td>Student is challenged to keep numbers and information in working memory</td>
<td>• Loses counts of objects</td>
<td>• Use ten-frames or organizational mats to help them organize counts.</td>
</tr>
<tr>
<td></td>
<td>• Gets too confused when multiple strategies are shared by other students during the after portion of the lesson</td>
<td>• Explicitly model how to use skip counting to count.</td>
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<td></td>
<td>• Forgets how to start the problem-solving process</td>
<td>• Jot down the ideas of other students during discussions.</td>
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<td></td>
<td>• Use ten-frames or organizational mats to help them organize counts.</td>
<td>• Incorporate a chart that lists the main steps in problem solving as an independent guide, or make bookmarks with questions the students can ask themselves as self-prompts.</td>
</tr>
<tr>
<td>Student lacks organizational skills and the ability to self-regulate</td>
<td>• Loses steps in a process</td>
<td>• Use routines as often as possible or provide self-monitoring checklists to prompt steps along the way.</td>
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<tr>
<td></td>
<td>• Writes computations in a way that is random and hard to follow</td>
<td>• Use graph paper to record problems or numbers.</td>
</tr>
<tr>
<td></td>
<td>• Use ten-frames or organizational mats to help them organize counts.</td>
<td>• Create math walls they can use as a resource.</td>
</tr>
<tr>
<td>Student misapplies rules or overgeneralizes</td>
<td>• Mechanically applies algorithms—for example, adds 7/8 and 12/13 and gives the answer</td>
<td>• Always give examples as well as counterexamples to show how and when “rules” should be used and when they should not.</td>
</tr>
<tr>
<td></td>
<td>• Always give examples as well as counterexamples to show how and when “rules” should be used and when they should not.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Tie all rules into conceptual understanding; don’t emphasize memorizing rote procedures or practices.</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 6.2 COMMON STUMBLING BLOCKS FOR STUDENTS WITH DISABILITIES**
Providing for Students with Special Needs

the number of words, illustrations, and problems on a page.

- **Vary the task size.** Assign students fewer tasks to solve to reduce frustration due to the enormity of the assignment.

- **Propose alternative products.** Provide options for how to demonstrate understanding (e.g., a dictated response that is written by someone else, an audio redacting of a verbal response, or a model made with a manipulative).

### Teaching Strategies

- **Systematic instruction.** Use repeated examples of the same problem, give repeated prompts, and provide corrective feedback.

- **Task chaining.** Take one step at a time on a mathematics task, giving a prompt for students at each step. Gradually fade the number of prompts based on student performance.

- **Response prompt.** Ask a student, “What is three plus three?” while visually showing $3 + 3$. If there is no response, say “Six” and then state to the student again, “Three plus three is six.” Next give a prompt and ask again, “What’s three plus three?”

- **Emphasize the relevant points.** Some students with special needs may focus on less relevant attributes, such as the colour of a cube instead of the quantity of cubes.

- **Visual supports.** Visual cues, colour coding, and simplified numerical expressions using dots or other pictorial clues can focus students’ learning.

- **Problem solving.** State the problem. For example, after passing out an insufficient number of paper plates, ask students, “What is the problem?” The students should state a solution and suggest that more materials are needed. “How many more plates are needed?” When that amount is given, students have solved the problem. Use a visual representation showing a one-to-one correspondence between people and plates to show how to record the situation. Then write and read the corresponding equation.

- **Self-determination skills and independent self-directed learning.** Support opportunities for students to make choices by decision making and goal setting.

### Students Who Are Mathematically Gifted and Talented

Children who are typically known as “gifted” also have special educational needs. They require appropriate educational opportunities and challenges to ensure that high expectations are set for them, as well. While children may be gifted in multiple areas, we focus here on mathematical interest and ability. Some may be gifted with an intuitive knowledge of mathematical concepts, whereas others have a passion for the subject even though they may have to work hard to learn it.

Although most provinces and territories use individual education programs (IEPs or IPPs) for students identified as gifted or high achieving, most schools districts do not have programs to screen for or identify all students, and
there is no federal legislation that mandates special pro-
grams for gifted students. In many cases, it is up to families
and teachers to actively advocate for opportunities for high-
ability students.

Many teachers have a keen ability to spot talent when
they note students who have strong number sense or visual/
spatial sense (Gavin & Sheffield, 2010). Note that these
teachers are not pointing to students who are fast and
speedy with their basic facts, but students who have the
ability to reason and make sense of mathematics. Having
gifted students in an inclusive classroom places additional
demands on the teacher that are different than those needed
to attend to struggling students. We start here with some of
the common, but ineffective strategies that find their way
into classrooms. Four common strategies to avoid are:

1. Assigning more of the same work. This is the least
appropriate way to respond to mathematically gifted
students and the most likely to result in students’ hid-
ing their ability. This approach is described by Persis
Herold as “all scales and no music” (quoted in Tobias,

2. Giving free time to early finishers. Although students
find this rewarding, it does not maximize their intel-
lectual growth and can lead to hurrying to finish a task.

3. Assigning gifted students to help struggling learners.
Routinely assigning gifted students to teach others
what they have mastered is an error in judgment,
because it puts mathematically talented students in a
constant position of tutoring rather than allowing them
to create deeper and more complex levels of under-
standing.

4. Independent enrichment on the computer. This prac-
tice does not engage students with mathematics in a
way that will enhance conceptual understandings and
support their ability to justify their thinking.

Now that you know what not to do, using Response to
Intervention strategies can support you with what you can
do. Response to Intervention is an appropriate support sys-
tem to use with students with both disability and ability. In
fact, gifted students may have additional special needs, such
as psychological and behavioural issues, that benefit from
the multi-tiered approach of RtI.

Creating challenging tasks and using tier 1 differentiation
strategies, including tiered tasks, learning centres, and
choice boards (discussed previously in this chapter) may be
highly successful for the specific needs of bright students.
A few additional tier 1 differentiation strategies for all stu-
dents, but with particular benefit to high achieving stu-
dents, include the following:

**Most Difficult First.** Teachers tend to assign the same
number of questions, tasks, or problems to all students
regardless of student needs. In many assignments with mul-
tiple questions, the tasks are arranged from the simplest to
the most complex and challenging. In this strategy, the
teacher identifies which questions are included as Most Dif-
cult First (MDF) questions. Students who demonstrate
proficiency in the most difficult problems need not com-
plete simpler tasks, as further repetition is not needed.

**Curriculum Compacting.** An extension of the most dif-
cult first strategy is curriculum compacting. This strategy
prevents re-teaching and re-teaching of content students
already know and understand. To compact, the teacher pre-
tests students in the content to be presented. Students mas-
tering, or nearly mastering, the content then move on to an
advanced level of difficulty (Reis & Renzulli, 2005). Teach-
ers can either reduce the amount of time these students
spend on aspects of the topic or move altogether to more
advanced and complex content. Moving students to higher
mathematics (by moving them up a grade, for example) will
not succeed in satisfying their learning needs because the
teaching of new material is likely to continue to be per-
ceived as too slow in pace and the student will continue to
teaching boredom. Gifted students should instead explore
similar topics as their classmates but focus on higher-level
thinking, more complex or abstract ideas, and deeper levels
of understanding. Research reveals that when gifted stu-
dents are accelerated through the curriculum they become
more likely to explore STEM fields (i.e., science, technol-
gy, engineering, and mathematics) (Sadler & Tai, 2007).

**Independent Study.** In independent study, students
research a teacher-selected or self-chosen topic, developing
either traditional or non-traditional products to demon-
strate their learning. This can mean exploring a larger set of
ideas in which a mathematics topic exists. For example,
while studying a unit on place value, mathematically gifted
students can stretch their knowledge to study other nume-
ration systems such as Roman, Mayan, Egyptian, Babylonian,
Chinese, and Zulu, providing a multicultural view of how
our number system fits within the number systems of
the world. In the algebra strand, when studying sequences
or patterns of numbers, mathematically gifted students
might learn about Fibonacci sequences and their appear-
ances in the natural world.

Generally, the assumption in education is that good
teaching is about being able to respond to the varying needs
of diverse learners, including the talented and gifted. Yet for
some gifted students, additional support is needed. Determin-
ing whether students need further support should not be
based on how they are doing relative to their peers, but
relative to their own abilities. For some students, strategic
targeted instruction at the tier 2 level is needed to ensure that
students are challenged or to address twice-exceptional—
intellectually gifted children with a disability—student
needs. Tier 2 strategies include supplemental mathematics
programs and targeted interventions aligned with tier 1
instruction.
Supplemental Mathematics Programs. Supplemental programs introduce completely different material from the regular curriculum and can frequently occur in after-school clubs, math competitions, out-of-class projects, or collaborative school experiences. Collaborative experiences may include students from a variety of grades and classes volunteering for special mathematics projects, with a classroom teacher, principal, or resource teacher taking the lead. This approach allows gifted students to explore topics that are within their developmental grasp but outside the curriculum. For example, students may look at mathematical “tricks” using binary numbers to guess classmates’ birthdays, or solve reasoning problems using a logic matrix. They may also explore topics such as topology through the creation of paper “knots” called flexagons (see www.flexagon.net) or large-scale investigations of the amount of food thrown away at lunchtime. A group might create tetrahedron kites or find mathematics in art. Another aspect of the novelty approach provides different options for students in culminating performances of their understanding, such as demonstrating their knowledge through inventions, experiments, simulations, dramatizations, visual displays, and oral presentations.

For students whose intellectual needs are not met by tier 1 and tier 2 strategies, longer-term individualized interventions are needed. Students who are highly gifted require more radical approaches to ensure that their learning needs are being met. Part-time and pull-out programs may not be enough. Students in tier 3 may require other means of acceleration, such as single-subject acceleration (if their talent in mathematics is significantly different than in other content areas), placement in classes that are several grades higher, mentorship with a mathematics specialist, or distance or online learning.

Final Thoughts

As you move into your own classroom, set high expectations for all students to succeed. The following general strategies support diversity in students’ languages, cultures, and abilities:
- Identify children’s current knowledge base and build instructions with that base in mind.
- Push all students to high-level thinking.
- Maintain high expectations.
- Use a multicultural approach.
- Recognize, value, explore, and incorporate the home culture.
- Use alternative assessments to broaden the variety of performance indicators.
- Measure progress over time rather than taking short snapshots of student work.
- Promote the importance of effort and resilience.

RESOURCES for Chapter 6

Recommended Readings

Articles


In this article, Alison Gear, an early-learning coordinator, describes her work with kindergarten students, parents, and grandparents, exploring mathematics from an Aboriginal perspective in a Haida community in British Columbia.


The articles in this focus issue address specific considerations for special students, strategies for differentiation, and more.


This issue is full of very useful articles and activities to support teachers working in diverse classrooms.


Books


This book includes information on teaching mathematics to students with disabilities by top mathematics educators. The chapters detail work in each of the five NCTM content strands as well as present models for developing a learning framework and assessing students.

Online Resources

Center for Applied Special Technology (CAST).

www.cast.org

This site contains resources and tools to support the learning of all students, especially those with disabilities, through Universal Design for Learning (UDL).
**Writing to Learn**

1. For children with learning disabilities and special learning needs, what are two strategies you can use to modify instruction?
2. Describe, in your own words, the central ideas of culturally relevant mathematics instruction.
3. What are some of the specific difficulties English language learners encounter in the mathematics class?

**For Discussion and Exploration**

1. What would you do if you found yourself teaching a class with one mathematically gifted student who had no equal in the room? Create a menu of six activities the student could consider on a mathematics topic of your choice. Include activities such as projects, data collection, games, integration with other content areas, links to literature, and/or complex problem solving (see Wilkins, Wilkins, & Oliver, 2006, for suggestions).

**MyEducationLab™**

Visit MyEducationLab to access an electronic version of the text, as well as a variety of topics that enhance the text material. The topics include the following to support your learning in the course:

- Assessment, including Building Teaching Skills and Dispositions and Video Assignments
- Discussion board questions
- Videos, simulations, activities, case studies, and other useful course resources