Credits and acknowledgments for material borrowed from other sources and reproduced, with permission, in this textbook appear on the appropriate page within the text.


If you purchased this book outside the United States or Canada, you should be aware that it has been imported without the approval of the publisher or the author.
About the Authors

John A. Van de Walle was a professor emeritus at Virginia Commonwealth University. He was a mathematics education consultant who regularly gave professional development workshops for K–8 teachers in the United States and Canada. He visited and taught in elementary school classrooms and worked with teachers to implement student-centered math lessons. He co-authored the Scott Foresman-Addison Wesley Mathematics K–6 series and contributed to the Pearson School mathematics program enVisionMATH. Additionally, he wrote numerous chapters and articles for the National Council of Teachers of Mathematics (NCTM) books and journals and was very active in NCTM, including serving on the board of directors, chairing the educational materials committee, and speaking at national and regional meetings.

Karen S. Karp is a professor of mathematics education at the University of Louisville (Kentucky). Prior to entering the field of teacher education she was an elementary school teacher in New York. Karen is a co-author of Feisty Females: Inspiring Girls to Think Mathematically, which is aligned with her research interests on teaching mathematics to diverse populations. With Jennifer, Karen co-edited Growing Professionally: Readings from NCTM Publications for Grades K–8 and co-authored (along with Janet Caldwell) Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Pre-K–Grade 2. She is a former member of the board of directors of the National Council of Teachers of Mathematics (NCTM) and a former president of the Association of Mathematics Teacher Educators (AMTE). She continues to work in classrooms with elementary and middle school teachers and with teachers at all levels who work with students with disabilities.

Jennifer M. Bay-Williams is a professor of mathematics education at the University of Louisville (Kentucky). Jennifer has published many articles on teaching and learning in NCTM journals. She has also co-authored numerous books, including Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Pre-K–Grade 2, Math and Literature: Grades 6–8, Math and Nonfiction: Grades 6–8, and Navigating Through Connections in Grades 6–8. Jennifer taught elementary, middle, and high school in Missouri and in Peru, and continues to work in classrooms at all levels with students and with teachers. Jennifer is on the board of directors for TODOS: Equity for All, is the editor for the 2012 NCTM Yearbook, and is a former president of the Association of Mathematics Teacher Educators (AMTE).
About the Contributor

Jonathan Wray is the technology contributor to Elementary and Middle School Mathematics: Teaching Developmentally. He is the instructional facilitator for secondary mathematics curricular programs in the Howard County public school system. He is the president elect of the Association of Maryland Mathematics Teacher Educators (AMMTE) and past president of the Maryland Council of Teachers of Mathematics (MCTM). He has been recognized for his expertise in infusing technology in mathematics teaching, receiving the Outstanding Technology Leader in Education award for his school district from the Maryland Society for Educational Technology (MSET). Jon is also actively engaged in NCTM, serving on the editorial panels of Teaching Children Mathematics and ON-Math. He has served as a primary and intermediate grades classroom teacher, gifted/talented resource teacher, elementary mathematics specialist, curriculum and assessment developer, grant project manager, and educational consultant.

About the Canadian Author

Lynn M. McGarvey is an associate professor of mathematics education at the University of Alberta. Lynn’s research focuses on the mathematical reasoning of young children. She has written many articles for research and professional audiences, particularly on the topics of algebraic thinking, spatial reasoning, and patterning in the early years. She is a long-time member of NCTM and has served on a number of task forces and committees, including as editorial panel chair for Teaching Children Mathematics. She is a former junior high school teacher who now spends a considerable amount of time working with children and teachers in preschools and kindergartens. Lynn has taught mathematics curriculum and pedagogy courses to thousands of pre-service and in-service elementary teachers and has won multiple teaching awards for this work.
Brief Contents

SECTION I
Teaching Mathematics: Foundations and Perspectives

Chapter 1  Teaching and Learning Mathematics in the Twenty-First Century  1
Chapter 2  Exploring What It Means to Know and Do Mathematics  11
Chapter 3  Mathematical Inquiry through Rich Tasks and Classroom Discourse  28
Chapter 4  Preparing to Teach and Planning for Mathematics Learning  45
Chapter 5  Blending Teaching and Assessment  64
Chapter 6  Teaching Mathematics for All Learners  84
Chapter 7  Tools for Learning Mathematics  103

SECTION II
Development of Mathematical Concepts and Procedures

Chapter 8  Developing Early Number Concepts and Number Sense  119
Chapter 9  Developing Meanings for the Operations  139
Chapter 10  Helping Students Master the Basic Facts  162
Chapter 11  Developing Whole-Number Place Value Concepts  183
Chapter 12  Developing Strategies for Addition and Subtraction Computation  208
Chapter 13  Developing Strategies for Multiplication and Division Computation  231
Chapter 14  Algebraic Thinking: Generalizations, Patterns, and Functions  253
Chapter 15  Developing Fraction Concepts  284
Chapter 16  Developing Strategies for Fraction Computation  310
Chapter 17  Developing Concepts of Decimals and Percents  333
Chapter 18  Proportional Reasoning  352
Chapter 19  Developing Measurement Concepts  369
Chapter 20  Geometric Thinking and Geometric Concepts  396
Chapter 21  Developing Concepts of Data Analysis  428
Chapter 22  Exploring Concepts of Probability  448
Chapter 23  Developing Concepts of Exponents, Integers, and Real Numbers  466

APPENDIX  Guide to Blackline Masters  485
REFERENCES  497
INDEX  517
Contents

Preface xvii

SECTION I
Teaching Mathematics: Foundations and Perspectives

The fundamental core of effective teaching of mathematics combines an understanding of how children learn, how to promote that learning by teaching through problem solving, and how to plan for and assess that learning on a daily basis. Introductory chapters in this section provide perspectives on trends in mathematics education and the process of doing mathematics. These chapters develop the core ideas of learning, teaching, planning, and assessment. Additional perspectives on mathematics for children with diverse backgrounds and the role of learning tools (e.g., manipulatives, technology) are also discussed.

CHAPTER 1
Teaching and Learning Mathematics in the Twenty-First Century 1

What Are Your Memories of School Mathematics? 1
Twenty-First Century Knowledge and Skills 2
Diversity in Today’s Classrooms 2
Mathematics Curriculum in Canada 3
   Goals for Student Learning 4
   Mathematics Content 4
   Mathematical Processes 5
An Invitation to Learn and Grow 7
   Becoming a Teacher of Mathematics 7
RESOURCES FOR CHAPTER 1
   Recommended Readings 9
   Online Resources 9
REFLECTIONS ON CHAPTER 1
   Writing to Learn 10
   For Discussion and Exploration 10

CHAPTER 2
Exploring What It Means to Know and Do Mathematics 11

What Does It Mean to Do Mathematics? 11
   A Classroom Environment for Doing Mathematics 12

An Invitation to Do Mathematics 12
Searching for Patterns 12
   Analyzing a Situation 14
   Generalizing Relationships 14
   Experimenting and Explaining 16
Engaging in Mathematical Inquiry 16
   Mathematics Proficiency 17
What Does It Mean to Learn Mathematics? 20
Mechanical Metaphors for Learning 20
   Learning Mathematics as Acquisition 21
   Learning Mathematics along a Linear Path 21
Ecological Metaphors for Learning 22
   Learning Mathematics as Coherence 23
   Learning Mathematics as Outward Growth 24
   An Example of Mathematics Learning 24
Learning Theories and Implications for Teaching 25
   Promoting Dispositions for Doing Mathematics 26
RESOURCES FOR CHAPTER 2
   Recommended Readings 26
   Online Resources 27
REFLECTIONS ON CHAPTER 2
   Writing to Learn 27
   For Discussion and Exploration 27
Why Mathematical Inquiry? 28
A Shift in the Role of Tasks in the Classroom 29
The Focus of Inquiry 30
What Is a Rich Task? 30
Features of a Rich Task 31
Variable Entry and Exit Points 31
Thinking Mathematically 32
High Levels of Cognitive Demand 34
Relevant Contexts 34
Evaluating and Adapting Tasks: What Do You Need to Do to Make Them Rich(er)? 36
Task Selection Guide 36
Developing Concepts and Procedures through Tasks 36
Concepts 36
Procedures 38
What about Drill or Practice? 38
New Definitions of Drill and Practice 38
What Practice Provides 39
What Drill Provides 39
When Is Drill Appropriate? 39
Drill and Student Errors 39
Creating a Culture of Mathematical Inquiry 40
Classroom Discourse 40
Questioning Considerations 42
How Much to Tell and Not to Tell 42
Preparing to Teach 43
RESOURCES FOR CHAPTER 3
Recommended Readings 43
Online Resources 44
REFLECTIONS ON CHAPTER 3
Writing to Learn 44
For Discussion and Exploration 44

Preparation to Teach and Planning for Mathematics Learning 45
A Three-Phase Lesson Format 45
Introduction to Inquiry: The Before Phase of a Lesson 45
Teacher Actions in the Before Phase 46
Development of Inquiry: The During Phase of a Lesson 48
Teacher Actions in the During Phase 48
Follow-Up to Inquiry: The After Phase of a Lesson 50
Teacher Actions in the After Phase 50
Process for Preparing a Lesson 51
1. Curriculum Learning Expectations 52
2. Student Experiences 52
3. Select, Design, or Adapt a Rich Task 53
4. Design the Lesson Assessments Tools 54
5. Plan the Before Phase of the Lesson 54
6. Plan the During Phase of the Lesson 55
7. Plan the After Phase of the Lesson 55
Final Revisions to Your Lesson Plan 56
Applying the Planning Process 56
Applying the Three-Phase Lesson Template to Other Lesson Structures 56
Applying the Three-Phase Model to Short Tasks 56
Applying the Three-Phase Model in Learning Centres 56
Homework and Parental Involvement 57
Effective Homework 58
Beyond Homework: Families Doing Math 58
Resources for Families 59
Frequently Asked Questions 59

RESOURCES FOR CHAPTER 4
Recommended Readings 60
Online Resources 61

REFLECTIONS ON CHAPTER 4
Writing to Learn 61
For Discussion and Exploration 61

EXPANDED LESSON  Fixed Area 62

Blending Teaching and Assessment 64
Purposes for Assessing Learning in Classrooms 64
What Should Be Assessed? 66
Mathematics Content 66
Mathematical Processes 67
Mathematical Disposition 67
What Tools Can Be Used to Gather Evidence of Mathematics Learning? 67
Performance-Based Tasks 68
Questions and Conversations 69
Interviews 69
Writing to Learn and Assess 70
Journals 72
Admit and Exit Slips 73
Inventories 74
Summative Assessment Tools 74
Cumulative Projects 74
SECTION II
Development of Mathematical Concepts and Procedures

This section serves as the application of the core ideas of Section I. Here you will find chapters on every major content area in the pre-K–8 mathematics curriculum. Numerous inquiry-based tasks and problems to engage students are interwoven with a discussion of the mathematical content and how children develop their understanding of that content. At the outset of each chapter, you will find a listing of “Big Ideas,” the mathematical umbrella for the chapter. Also included are ideas for incorporating children’s literature, technology, and assessment. These chapters are designed to help you develop pedagogical strategies and to serve as a resource for your teaching now and in the future.

CHAPTER 8
Developing Early Number Concepts and Number Sense

Promoting Good Beginnings 119
Number Concepts: Quantity, Counting, and Knowing How Many 120
  Quantity and the Ability to Subitize 121
  Early Counting 122
  Numeral Writing and Recognition 123
  Counting On and Counting Back 124
Comparing Sets: More Than, Less Than, and Equal To 125
Early Number Sense 126
Relationships for Numbers 1 through 10 127
  One and Two More, One and Two Less 127
  Anchoring Numbers to 5 and 10 128
  Part-Part-Whole Relationships 130
  Dot Cards as a Model for Teaching Number Relationships 133
Relationships for Numbers 10 through 20 134
  Early Place Value Concepts 134
  Extending More-Than and Less-Than Relationships 134
Number Sense in Their World 135
  Estimation and Measurement 135
  Data Collection and Analysis 136
  Classroom Routines 136
RESOURCES FOR CHAPTER 8
  Literature Connections 137
  Recommended Readings 137
  Online Resources 138
REFLECTIONS ON CHAPTER 8
  Writing to Learn 138
  For Discussion and Exploration 138

CHAPTER 9
Developing Meanings for the Operations

Addition and Subtraction Problem Structures 140
  Join and Separate Problems 140
  Part-Part-Whole Problems 141
  Compare Problems 141
  Problem Difficulty 142
  Computational and Semantic Forms of Equations 142
Teaching Addition and Subtraction 142
  Contextual Problems 142
MATH MAKES SENSE 144
  Model-Based Problems 145
  Properties of Addition and Subtraction 147
Multiplication and Division Problem Structures 148
  Equal Groups Problems 148
  Comparison Problems 149
  Area and Array Problems 149
  Combination Problems 151
Teaching Multiplication and Division 151
  Contextual Problems 151
  Remainders 152
  Model-Based Problems 153
  Properties of Multiplication and Division 155
Strategies for Solving Contextual Problems 156
  Analyzing Context Problems 156
  Two-Step Problems 158
RESOURCES FOR CHAPTER 9
  Literature Connections 159
  Recommended Readings 159
  Online Resources 160
REFLECTIONS ON CHAPTER 9
  Writing to Learn 160
  For Discussion and Exploration 160
CHAPTER 10
Helping Students Master the Basic Facts 162

Developmental Nature of Basic Fact Mastery 162
  Approaches to Fact Mastery 163
  Guiding Strategy Development 164
Reasoning Strategies for Addition Facts 165
  One More Than and Two More Than 166
  Adding Zero 167
  Using 5 as an Anchor 167
  Make 10 167
  Up Over 10 167
  Doubles 168
  Near-Doubles 169
  Reinforcing Reasoning Strategies 169
Reasoning Strategies for Subtraction Facts 170
  Subtraction as Think-Addition 170
  Down Over 10 171
  Take from the 10 171
Reasoning Strategies for Multiplication and Division Facts 172
  Doubles 172
  Fives 172
  Zeros and Ones 172
  Nifty Nines 173
  Using Known Facts to Derive Other Facts 174
  Division Facts 175
Mastering the Basic Facts 175
  Effective Drill 175
  Games to Support Basic Fact Mastery 176
Fact Remediation 177
  What to Do When Teaching Basic Facts 179
  What Not to Do When Teaching Basic Facts 179

RESOURCES FOR CHAPTER 10
  Literature Connections 180
  Recommended Readings 180
  Online Resources 181

REFLECTIONS ON CHAPTER 10
  Writing to Learn 181
  For Discussion and Exploration 182

CHAPTER 11
Developing Whole-Number Place Value Concepts 183

Numeration System 183
  Pre-Base-Ten Understandings 184
    Counting by Ones 184

Basic Ideas of Place Value 184
  Integration of Base-Ten Groupings with Counting by Ones 185
  Role of Counting 185
  Integration of Groupings with Words 186
  Integration of Groupings with Place Value Notation 186

Base-Ten Models for Place Value 187
  Groupable Models 187
  Pregrouped or Trading Models 187
  Nonproportional Models 188

Developing Base-Ten Concepts 188
  Grouping Activities 188
  The Strangeness of Ones, Tens, and Hundreds 189
  Grouping Tens to Make 100 190
  Equivalent Representations 191

Oral and Written Names for Numbers 192
  Two-Digit Number Names 192
  Three-Digit Number Names 193
  Written Symbols 194
  Assessing Place Value Concepts 195

Patterns and Relationships with Multidigit Numbers 195
  The Hundreds Chart 195

Relationships with Benchmark Numbers 197
  Connecting Place Value to Addition and Subtraction 198
  Connections to Real-World Ideas 202

Numbers beyond 1000 203
  Extending the Place Value System 203
  Conceptualizing Large Numbers 204

RESOURCES FOR CHAPTER 11
  Literature Connections 205
  Recommended Readings 206
  Online Resources 206

REFLECTIONS ON CHAPTER 11
  Writing to Learn 206
  For Discussion and Exploration 207

CHAPTER 12
Developing Strategies for Addition and Subtraction Computation 208

Toward Computational Fluency 209
  Student-Generated (Personal) Strategies 211
  Standard Algorithms 212

Development of Meaningful Strategies 214
  Creating an Environment for Meaningful Strategies 214
  Models to Support Meaningful Strategies 214

Student-Generated Strategies for Addition and Subtraction 217
  Adding and Subtracting Single-Digit Numbers 217
  Adding Two-Digit Numbers 218
Contents

Subtracting by Counting Up 219
Take-Away Subtraction 219
Extensions and Challenges 220

Algorithms for Addition and Subtraction 221
  Standard Algorithm for Addition 221
  Alternative Algorithms for Addition 223
  Standard Algorithm for Subtraction 223
  Alternative Algorithms for Subtraction 224

Introducing Computational Estimation 225
  Understanding Computational Estimation 225
  Suggestions for Teaching Computational Estimation 225

Computational Estimation Strategies 227
  Front-End Methods 227
  Rounding Methods 227
  Compatible Numbers 228

RESOURCES FOR CHAPTER 12
  Literature Connections 229
  Recommended Readings 229
  Online Resources 230

REFLECTIONS ON CHAPTER 12
  Writing to Learn 230
  For Discussion and Exploration 230

CHAPTER 13
Developing Strategies for Multiplication and Division Computation 231

Towards Computational Fluency with Multiplication and Division 232
Development of Meaningful Strategies for Multiplication and Division 232
  Models to Support Meaningful Strategies 232
Student-Generated Strategies for Multiplication 235
  Complete-Number Strategies (Including Doubling) 236
  Multiplication of Larger Numbers 236
Standard Algorithms for Multiplication 238
  One-Digit Multipliers 238
  Two-Digit Multipliers 239
  Alternative Algorithms for Multiplication 240
Student-Generated Strategies for Division 240
Missing-Factor Strategies 240
  Cluster Problems 241
Standard Algorithms for Division 242
  One-Digit Divisors 242
  Alternative Division Algorithms 244
Computational Estimation in Multiplication and Division 245
  Understanding Computational Estimation 245
  Suggestions for Teaching Computational Estimation 245
   Computational Estimation from Student-Generated Strategies 246
      Stop before the Details 246
      Use Related Problem Sets 246
Computational Estimation Strategies 247
  Front-End Methods 247
  Rounding Methods 247
  Compatible Numbers 248
  Using Tens and Hundreds 248
Estimation Experiences 249
  Calculator Activities 249
  Using Whole Numbers to Estimate Rational Numbers 251

RESOURCES FOR CHAPTER 13
  Literature Connections 251
  Recommended Readings 252
  Online Resources 252

REFLECTIONS ON CHAPTER 13
  Writing to Learn 252
  For Discussion and Exploration 252

CHAPTER 14
Algebraic Thinking: Generalizations, Patterns, and Functions 253

Algebraic Thinking 254
  Generalization from Arithmetic 254
    Generalization with Operations 254
    Generalization in the Hundreds Chart 255
    Generalization through Exploring a Pattern 256
  Meaningful Use of Symbols 257
    The Meaning of the Equal Sign 257
    The Meaning of Variables 262
  Making Structure in the Number System Explicit 265
    Making Conjectures about Properties 265
    Justifying Conjectures 267
    Odd and Even Relationships 267
  Study of Patterns and Functions 267
    Repeating Patterns 268
    Growing Patterns 270
    Linear Functions 274
  Mathematical Modelling 276
  Teaching Considerations 277
    Emphasize Appropriate Algebra Vocabulary 277
    Connecting Representations 278
    Algebraic Thinking across the Curriculum 280

RESOURCES FOR CHAPTER 14
  Literature Connections 281
  Recommended Readings 282
  Online Resources 282
CHAPTER 21
Developing Concepts of Data Analysis 428

What Does It Mean to Do Statistics? 429
Is It Statistics or Is It Mathematics? 429
The Shape of Data 429
The Process of Doing Statistics 430

Formulating Questions 430
Classroom Questions 430
Beyond One Classroom 431

Data Collection 431
Collecting Data 431
Using Existing Data Sources 432

Data Analysis: Classification 432
Attribute Materials 433

Data Analysis: Graphical Representations 434
Bar Graphs and Tally Charts 435
Circle Graphs 436
Numerical Data Graphs 437
Scatter Plots 439

Data Analysis: Measures of Centre and Variability 440
Averages 440
Understanding the Mean: Two Interpretations 441
Variability 443
Box Plots 444

Interpreting Results 445

RESOURCES FOR CHAPTER 21
Literature Connections 446
Recommended Readings 446
Online Resources 446

REFLECTIONS ON CHAPTER 21
Writing to Learn 447
For Discussion and Exploration 447

CHAPTER 22
Exploring Concepts of Probability 448

Introducing Probability 448
Likely or Not Likely 449
The Probability Continuum 450

Theoretical Probability and Experiments 453
Theoretical Probability 453
Experiments 454
Why Use Experiments? 456
Use of Technology in Experiments 456

Sample Spaces and Probability of Two Events 457
Independent Events 457
Area Models 459
Dependent Events 460

Simulations 461

RESOURCES FOR CHAPTER 22
Literature Connections 463
Recommended Readings 463
Online Resources 464

REFLECTIONS ON CHAPTER 22
Writing to Learn 464
For Discussion and Exploration 464

CHAPTER 23
Developing Concepts of Exponents, Integers, and Real Numbers 466

Exponents 466
Exponents in Expressions and Equations 466
Order of Operations 467
Negative Exponents 470
Scientific Notation 470

Integers 472
Contexts for Exploring Integers 472
Quantity Contexts 472
Linear Contexts 473
Meaning of Negative Numbers 474
Models for Teaching Integers 474
Contents

Operations with Integers 475
  Addition and Subtraction 475
  Multiplication and Division 478

Real Numbers 478
  Rational Numbers 479
  Square Roots and Cube Roots 480

RESOURCES FOR CHAPTER 23
  Literature Connections 482
  Recommended Readings 482
  Online Resources 482

REFLECTIONS ON CHAPTER 23
  Writing to Learn 482
  For Discussion and Exploration 483

APPENDIX
  Guide to Blackline Masters 485

References 497

Index 517
New to This Edition

The fourth Canadian edition has been revised to include the following changes to better prepare teachers to teach mathematics to Canadian learners:

- **Section I (Chapters 1–7)** has been substantially revised to emphasize and describe mathematics teaching, learning, and curriculum across Canada. In particular, twenty-first century knowledge and skills including problem solving, creativity, collaboration, communication, and social responsibility are introduced.

- In the third edition, Chapter 1 was based substantially on two aspects: NCTM Principles and Standards for School Mathematics (NCTM, 2000), and provincial, national, and international testing programs. Chapter 1 now examines the curriculum documents of the provinces and territories across Canada and notes their similarities and differences in terms of goals for student learning, mathematics content, and mathematical processes.

- Diverse learners are also introduced in Chapter 1 as diversity is the norm in most Canadian classrooms. Diverse learning needs are described here and throughout the text as opportunities to enhance learning for all students rather than as something to eliminate. The chapter explicitly addresses First Nations, Métis, and Inuit students, English language learners, and learners with special needs including those with cognitive, behavioural, and physical exceptionalities.

- Chapter 2 is substantially revised to expand on the nature of mathematics—from being the science of patterns or as problem solving to a more encompassing perspective that mathematics is a way of thinking and seeing. The tasks within all the chapters are revised to reflect this expanded viewpoint. Canadian examples are used throughout. The Chapter 2 section on learning theory is also revised. Rather than singling out specific learning theories, the emphasis is on helping teachers recognize widespread beliefs about learning and practices reflected in the metaphors we use (e.g., “I'm falling behind”).

- The first two chapters set the stage for revisions in Chapter 3. Its previous focus on teaching through problem solving now takes a broader focus on mathematical inquiry through rich tasks and classroom discourse. A more in-depth discussion is provided that outlines the features of a rich task using a multitude of examples. The discussion surrounding tasks such as drill and homework have been incorporated into Chapter 3. Problem solving is considered one form of inquiry alongside other types of worthwhile tasks. Given this shift in focus, mathematical habits of mind replace problem solving strategies to encompass broader perspectives of mathematical thinking and doing.

- In previous editions, lesson preparation and planning based on a three-phase lesson format was been spread across two chapters. In this fourth Canadian edition, preparation and planning are placed together in Chapter 4, entitled “Preparing to Teach and Planning for Mathematics Learning.”

- Chapter 5 has been revised to use the framework of assessment for learning, as learning, and of learning throughout, using the questions: For what purpose? Who is responsible? What is assessed? What tools are used? and How is assessment data recorded? Substantial attention is paid to assessment of content, processes, and dispositions within Canadian
Many more examples of children's work and assessment tools have been added to this chapter.

- **Chapter 6**, now Teaching Mathematics for All Learners, has also been substantially revised. In the previous edition, the chapter focused extensively on specific disabilities and general ways to address specific learning needs. The current chapter has been revised to focus on differentiated teaching and learning; in particular, it builds on criteria for rich tasks from Chapter 3 to create tiered tasks, learning centres, choice boards, and so on. Diversity within the classroom is also addressed in more detail and takes into account different cultural ways of learning, culturally responsive teaching strategies, and linguistic diversity. Students with special needs are addressed in more detail through the framework of Response to Intervention, which offers a way to consider a tiered student support system and forms of intervention.

- The focus for Chapter 7, which was previously on technology and school mathematics, has been expanded to address Tools for Learning Mathematics, of which technology is one type. The chapter is based on three types of tools used in Canadian curriculum, including (1) concrete materials and physical models (e.g., manipulatives); (2) visual and graphic representations (e.g., pictures, diagrams, schematic drawings, graphs); and (3) technology-based tools (e.g., calculators, dynamic geometry software). The chapter provides details on how tools can be used appropriately in the classroom to enhance learning.

Section II of the Textbook has been substantially updated with new research, along with a number of new features.

- **New adaptations and accommodations for diverse learners**, including English language learners and students with difficulties in mathematics, appear not only in the narrative in Section I but also in many activities through direct examples and descriptions for the various content areas in Section II. The increased emphasis on diversity will be obvious to those who have used the book in the past.

- **New samples of authentic student work** illustrate student thinking. Student responses present glimpses into how students think about problems and what students' written work on mathematical tasks looks like, increasing teachers' awareness of how rich students' mathematical thinking can be—and how high our expectations should be.

- **Increased early childhood coverage** provides expanded emphasis on and reorganization of early numeracy in Chapters 8 and 9. Based on learning for the areas of number, relations, and operations, the work with early learners is seen as the essential foundation for number sense and mathematical ways of thinking.

- **New Formative Assessment Notes** in each chapter in Section II guide readers through ideas they can test with individual students or students in groups. Formative assessment is one of the key tools in finding out what students are thinking, and thereby identifying their areas of strength and weakness. Chapter 5 contains a more detailed description of formative assessments organized in three major assessment areas: tasks, observation, and interviews. To bring these ideas to life and to make them more directly linked to the content, these Formative Assessment Notes are included throughout the content area chapters to support teachers in the effective use of formative assessment, which is directly connected to increased student achievement.

- **Extensively updated information on how to effectively integrate new technological tools** to support teaching and learning appears in select Activities throughout the text.

- **A reorganization of Chapters 12 and 13** emphasizes both strategies for computation and estimation for addition and subtraction in Chapter 12, and the same for
multiplication and division in Chapter 13. This is a change from the third edition, which separated developing strategies for whole number computation and estimation for the four operations.

- A discussion on engaging families in meaningful ways to help students learn mathematics appears in Chapter 4.
- Additional attention to classroom discourse now appears in Chapter 3, Mathematical Inquiry through Rich Tasks and Classroom Discourse. The coverage includes how to conduct productive discussion sessions and develop effective questioning, and is illustrated with a vignette.

Other Changes of Note

Much has changed on the landscape of mathematics education, and so many aspects of the book have been updated to reflect those changes. In addition to the changes listed above, the following substantive changes have been made:

- There is an increased focus on the research-based developmental model of developing basic facts, and new activities to support basic fact mastery appear in Chapter 10.
- The content on algebraic thinking has been adapted to align with current research and changes to curricula across Canada. There is an increased emphasis on equivalence and variables, including adding the number-line representation of variables and making the properties more explicit.
- Chapter 15 (Developing Fraction Concepts) has greatly expanded sections on partitioning and on equivalence to reflect three recent research reviews that have indicated that this is essential to all advanced fraction work and success in algebra.
- Chapter 16 (Developing Strategies for Fraction Computation) now includes Activities—10 new ideas for developing understanding of fraction operations.
- Chapter 18 has been shortened, had new activities added, and been refocused to address understanding of ratios more deeply (with less focus on connecting to other content areas).
- The chapter on measurement, Chapter 19, has been reorganized. Previously, the development of all measurement formulas was shared at the end of the chapter; now, the formulas are integrated with the corresponding measurement topic (e.g., area or volume). The content has also been revised to ensure the focus is on metric measurement.
- Chapter 21 gives more explicit attention to distinguishing between numerical data and categorical data.
- Chapter 23 includes a significantly revised section on order of operations and numerous new activities.

What You Will Find in This Book

When you look at the table of contents, you will see that the chapters are separated into two distinct sections. The first section, consisting of seven chapters, deals with important ideas that cross the boundaries of specific areas of content. The second section, consisting of 16 chapters, offers teaching suggestions and activities for every major mathematics topic in the pre-kindergarten to Grade 8 curriculum. Chapters in Section I offer perspectives on the challenging task of helping students learn mathematics. Having a feel for the discipline of mathematics—that is, to know what it means to “do mathematics”—is critical to learning how to teach mathematics well. In addition, understanding perspectives on learning
mathematics and how different perspectives reflect different approaches to teaching provides a foundation and rationale for how to teach and assess pre-K–8 students.

Importantly, you will be teaching diverse students, including students who are English language learners, are gifted, and/or have difficulties learning. You will learn how to apply instructional strategies in ways that support and challenge all learners. Formative assessment, strategies for diverse learners, and effective use of learning tools are addressed in specific chapters in Section I (Chapters 5, 6, and 7, respectively), and throughout the Section II chapters.

Each chapter of Section II focuses on one of the major content areas in pre-K–8 mathematics curriculum. It begins with identifying the big ideas for that content, then provides guidance on how students best learn that content, along with many worthwhile tasks to engage students in understanding mathematics. Reflecting on the activities as you read can help you think about the mathematics from the perspective of the student. As often as possible, take out pencil and paper and try the problems so that you actively engage in your learning about students learning mathematics. We hope this book will increase your own understanding of mathematics, of the students you teach, and of how to teach them well.

### Some Special Features of this Text

Upon flipping through the book, you will notice many section headings, a large number of figures, and various special features. All are designed to make the book more useful as a textbook and as a long-term resource. Here are a few things to look for.

#### Big Ideas

Much of the current research and literature in mathematics education suggests that teachers plan their instruction around “big ideas” rather than isolated skills or concepts. At the beginning of each chapter in Section II, you will find a list of the key mathematical ideas associated with the chapter. Teachers find these lists helpful for quickly getting a picture of the mathematics they are teaching.

#### Mathematics Content Connections

Following the Big Ideas lists are brief descriptions of other content areas in mathematics that are related to the content of the current chapter. These lists are offered to help you be more aware of the potential interaction of content as you plan lessons, diagnose students’ difficulties, and learn more about the mathematics you are teaching.
Technology Ideas

Technology is an important tool for learning mathematics, as you will learn in Chapter 7. We have infused technology ideas throughout Section II. An icon is used to identify those places within the text or an activity where a technology idea or resource is discussed. Descriptions include open-source (free) software, applets, and other Web-based resources, as well as calculator ideas.

Formative Assessment Notes

Assessment should be an integral part of instruction, so it makes sense to think about what to be listening for (assessing) as you read about different areas of content development. Throughout the content chapters, you will see Formative Assessment Note icons indicating a short description of ways to assess the topic in that section. Reading these assessment notes as you read the text can help you understand how best to help your students.

Activities

The numerous activities found in every chapter of Section II have always been rated by readers as one of the most valuable parts of the book. Some activity ideas are described directly in the text and in the illustrations. Others are presented in the numbered Activity boxes. Every activity is a problem-based task (as described in Chapter 3) and is designed to engage students in doing mathematics. New adaptations and accommodations for diverse learners and differentiation strategies for students with difficulties are included in many activities.

Activity 17.2

Base-Ten Fractions to Decimals

For this activity, have students use their paper place value strips and squares (Bladeline Master 1-4). Agree that a large square represents 1. Have students cover a base-ten fractional amount of the square using their strips and times (remember to call the pieces "tenths" and "hundredths"). For example, have them cover 2/3 of the square. Whole numbers require additional squares. The task is to decide how to write and say this fraction as a decimal and demonstrate the connection using their physical models. For students with difficulties, you may want to have the amount shaded rather than have the students try to cover the exact amount; then ask them to name and write the decimal fraction.

Activity 17.3

Calculator Decimal Counting

Recall how to make the calculator "count" by pressing 1, 2, 3, 4, 5, 6, 7, 8, 9. Now have students press 1, 2, 3, 4, 5, 6, 7, 8, 9. When the display shows 0.0, stop and discuss what this means and what the display will look like with the next count. Many students will predict 0.10 (thinking that 0.10 is the next counting number). This prediction is correct if, with each press, the students have been accumulating base-ten strips as models for tenths. One more press would mean one more strip, or 10 strips. Why then the calculator shows 0.010? When the tenth press produces a display of 1 (calculators are not usually set to display trailing zeros to the right of the decimal), the discussion should revolve around trading 10 strips for a square. Continue to count to 4 or 5 by tenths. How many
Exploration.

For Discussion and Exploration

These questions ask you to explore an issue related to that student learning.

Literature Connections

Section II chapters contain examples of great children’s literature for launching into the mathematics concepts in the chapter just read.

Recommended Readings

In this section, you will find an annotated list of articles and books to augment the information found in the chapter.

Online Resources

At the end of each chapter, you will find an annotated list of some of the best Web-based resources along with their website addresses so that you can further explore how to infuse technological tools into instruction to support student learning.

Writing to Learn

Questions are provided that help you reflect on the important pedagogical ideas related to the content in the chapter.

For Discussion and Exploration

These questions ask you to explore an issue related to that chapter, applying what you have learned.
Appendix

The Appendix contains images of all of the Blackline Masters (BLM) that are listed below. The blackline masters can be found on the MyEducationLab website (www.myeducationlab.com).

### Expanded Lessons

An example of an Expanded Lesson can be found at the end of Chapter 4. In addition, eight similar Expanded Lessons can be found on MyEducationLab at www.myeducationlab.com.

#### Очевидно, существо задачи остигнуто. Формат и структура документа сохранены в естественном виде, с учетом дополнительных заметок и ссылок.
Supplements

MyEducationLab

MyEducationLab for Curriculum and Instruction (www.myeducationlab.com) is an online learning solution that provides interactive exercises designed to help teacher candidates develop the knowledge and skills that teachers need. Using classroom video, authentic student and teacher artifacts, and other resources and assessments, the learning experiences in MyEducationLab offer you a unique and valuable education tool.

For each mathematics topic covered, you will find the following features and resources:

- **Expanded Lesson Activities**: these expanded activities will help you explore how to use the lessons in the classroom.
- **Artifact Analysis Activities**: activities contain audio and visual records of students interacting with mathematics, accompanied by questions that focus on the analysis of student thinking.
- **Activities for Developing Mathematics Content Knowledge**: designed to assess and develop your mathematics content understanding, activities focus on areas where people typically have misconceptions.
- **Building Teaching Skills and Dispositions**: learning units that help teacher candidates practise and strengthen skills that are essential to effective teaching.
- **Video Assignments**: a robust selection of classroom videos accompanied by a set of questions.
- **IRIS Modules and Case Studies**: course-enhancement materials from the IRIS Center, designed to better prepare school personnel to provide an appropriate education to students.
- **Lesson Plan Builder**: an easy-to-use tool you can use to create, update, and share quality lesson plans.

Pearson eText

Pearson eText gives students access to the text whenever and wherever they have access to the Internet. eText pages look exactly like the printed text, offering powerful new functionality for students and instructors. Users can create notes, highlight text in different colours, create bookmarks, zoom, click hyperlinked words and phrases to view definitions, and view in single-page or two-page view. Pearson eText allows for quick navigation to key parts of the eText using a table of contents, and provides full-text search. The eText may also offer links to associated media files, enabling users to access videos, animations, or other activities as they read the text.

CourseSmart for Students

CourseSmart goes beyond traditional expectations—providing instant, online access to the textbooks and course materials you need at an average savings of 60%. With instant access from any computer and the ability to search your text, you’ll find the content you need quickly, no matter where you are. And with online tools like highlighting and note-taking, you can save time and study efficiently. See all the benefits at www.coursesmart.com/students.

For Instructors

The following instructor’s supplements are available for downloading from a password-protected section of Pearson Education Canada’s online catalogue: www.pearsoncanada.ca/highered. Navigate to your book’s catalogue page to view a list of those supplements that are available. See your local sales representative for details and access.

Instructor’s Manual  The Instructor’s Manual includes a wealth of resources designed to help instructors teach the course, including chapter notes, activity suggestions, suggested assessments, and test questions.

PowerPoint Presentation  Ideal for instructors to use for lecture presentations or student handouts, the PowerPoint presentation provides dozens of ready-to-use graphic and text images tied to the text.

CourseSmart for Instructors

CourseSmart goes beyond traditional expectations—providing instant, online access to the textbooks and course materials you need at a lower cost for students. And even as students save money, you can save time and hassle with a digital eTextbook that allows you to search for the most relevant content at the very moment you need it. Whether it’s evaluating textbooks or creating lecture notes to help students with difficult concepts, CourseSmart can make life a little easier. See how when you visit www.coursesmart.com/instructors.

Pearson Custom Library

For enrollments of at least 25 students, you can create your own textbook by choosing the chapters that best suit your own course needs. To begin building your custom text, visit www.pearsoncustomlibrary.com. You may also work with a dedicated Pearson Custom editor to create your ideal text—publishing your own original content or mixing and matching Pearson content. Contact your local Pearson Representative to get started.

Learning Solutions Managers

Pearson’s Learning Solutions Managers work with faculty and campus course designers to ensure that Pearson
technology products, assessment tools, and online course materials are tailored to meet your specific needs. This highly qualified team is dedicated to helping schools take full advantage of a wide range of educational resources, by assisting in the integration of a variety of instructional materials and media formats. Your local Pearson Education sales representative can provide you with more details on this service program.

Acknowledgments from the US Edition

Many talented people have contributed to the success of this book, and we are deeply grateful to all those who have assisted over the years. Without the success of the first edition, there would certainly not have been a second, much less eight editions. John worked closely with Warren Crown, John Dossey, Bob Gilbert, and Steven Willoughby, who gave time and great care in offering detailed comments on the original manuscript.

In preparing the eighth US edition, we have received thoughtful input from the following educators who offered comments on the seventh edition or on the manuscript for the eighth:

Margaret Adams, University of North Carolina at Charlotte
Joohi Lee, University of Texas at Arlington
Sandra J. Phifer, Metropolitan State College of Denver
Diana Piccolo, Missouri State University
Janet Lynne Tassell, Western Kentucky University

Each reviewer challenged us to think through important issues. Many specific suggestions have found their way into this book, and their feedback helped us focus on important ideas. We are indebted to these committed professionals.

We also would each like to thank our families for their many contributions and support. On behalf of John, we thank his wife of more than 40 years, Sharon. Sharon was John’s biggest supporter in this process and remained a sounding board for his many decisions as he wrote the first six editions of this book. We also thank his daughters, Bridget (a fifth-grade teacher in Chesterfield County, Virginia) and Gretchen (an associate professor of psychology at Rutgers University–Newark). They were John’s first students and he tested many ideas that are in this book by their sides. We can’t forget those who called John “Math Grandpa”: his granddaughters, Maggie, Aidan, and Gracie.

From Karen Karp: I would like to express thanks to my husband, Bob Ronau, who as a mathematics educator graciously helped me think about decisions while offering insights and encouragement. In addition, I thank my children, Matthew, Tammy, Joshua, Misty, Matt, Christine, Jeffrey, and Pamela for their kind support and inspiration.

I also am grateful for my wonderful grandchildren, Jessica, Zane, and Madeline, who have helped deepen my understanding about how children think.

From Jennifer Bay-Williams: I am so grateful to my husband, Mitch, who offers support, guidance, and wisdom to my writing, and my children, MacKenna (8 years) and Nicolas (6 years), who enjoy doing a little extra math from time to time. My parents, siblings, and nieces and nephews have all contributed ideas and support to the writing of this edition. Finally, I want to thank Brandy Jones, who has been invaluable in helping me find research to inform my writing for this edition.

Most importantly, we thank all the teachers and students who gave of themselves by assessing what worked and what didn’t work in the many iterations of this book. In particular for the eighth edition, we thank teachers who generously tested activities and provided student work for us: Kyle Patterson, Kim George, Kelly Eaton, Sarah Bush, and Elizabeth Popelka. If future teachers learn how to teach mathematics from this book, it is because teachers and children before them shared their best ideas and thinking with the authors.

Acknowledgements from the Canadian Edition

From Lynn McGarvey: I am indebted to my husband, Don, and my son, Matthew, who have been incredibly patient and supportive throughout the many phases of this project. I need to thank the creative and inspiring instructors who have been part of the Mathematics Subject Area Committee at the University of Alberta. These educators are constantly expanding my understanding of mathematics teaching and learning: Jane Gannon, Audrey Hodgson-Ward, Bev Kula, Julie Long, Janelle McFeeters, Gladys Sterenberg, and Carrie Watt. Finally, my contribution to the textbook has been with my students in mind. They have been instrumental in challenging me to connect theory, practice, and their personal histories to what it means to think mathematically.

In preparing this fourth Canadian edition, the following educators were among those who offered helpful and insightful comments on the previous edition and the draft manuscript:

Jerry Ameis, University of Winnipeg
Jayne Asselin, Fleming College
David Bale, University of Regina
Norma Fraser, Thompson Rivers University
Nahid Golafshani, Laurentian University
Audrey Hodgson-Ward, University of Alberta
Harpreet Kaur, Simon Fraser University
Wendy Klassen, UBC Okanagan
Jo-Anne Lake, Queen’s University
Annie Savard, McGill University
Gladys Sterenberg, University of Alberta
David Watson, Western University